Organizational versus geometric factors in mental rotation and folding tasks

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Abstract. The criteria used in performing mental rotation or mental folding tasks were studied with a paradigm that did not involve reaction times. The hypothesis was that, when perceptual-organizational factors come into conflict with the geometric features required for the correct execution of such tasks, it is the former that prevail. To verify this hypothesis two experiments were carried out. In experiment 1, subjects were asked to imagine quadrilaterals rotating round a rotation axis at different inclinations. Their responses were dependent both on the degree of tilt of the rotation axis and on the degree of tilt of the quadrilateral with respect to the rotation axis. Experiment 2 consisted of the mental execution of a folding task. In this case too, the responses depended on the degree of tilt of the folding axis and also on the complexity of the stimulus outline. In both experiments responses were divided into two groups: (i) geometrically correct responses and (ii) responses which, although incorrect, were based on perceptual-organizational criteria. In the light of the results, some theoretical implications regarding transformation operations executed by means of mental images are discussed.

1 Introduction

Perceptual activity provides us not only with information useful for our successful interaction with object surrounding us, but may also be the foundation on which we base our *conceptions* and *mental operations* of *restructuring, reassembling, projecting,* or transforming objects (Block 1981; Cooper and Shepard 1978; Kosslyn 1980; Kosslyn and Pomerantz 1977; Paivio 1971, 1975; Shepard and Cooper 1982). Many studies have dealt with the difference and relative independence between the perceptual moment at which information is collected, and the inferential or conceptual moment at which that information is used to solve problems: representing, respectively, the 'primary' and 'secondary' processes (see Kanizsa 1979; a similar interpretation can be attributed to the 'levels' of perceptual activity in Dodwell 1975). To demonstrate this separation between perception and conception, many different conditions have been described in which there is a perceptual result without logic or knowledge of this effect (Kanizsa 1985; Kanizsa and Luccio 1986, 1987). Naturally, inferential and perceptual processes do not always proceed independently of each other; they eventually become integrated when faced with complex tasks.

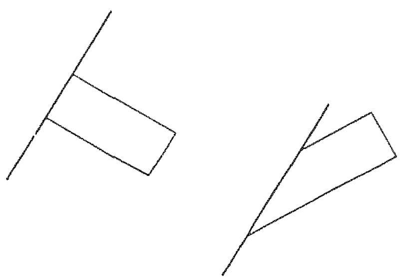
In operations that involve handling of geometric spatial elements, there may be cases in which organizational factors of the figural kind come into conflict with the physical-geometric aspects necessary for correct task performance. As Perkins (1983) has already noted, the human visual system is generally a "good organizer" but a "bad geometer". In other words, in man (unlike what happens in artificial vision) physical-geometric factors usually succumb when they conflict with organizational factors.

Shepard and his co-workers have studied the spatial transformation of visual objects through mental images (see Shepard and Cooper 1982). Because the experimental paradigm of reaction time (RT) was mainly used in these studies, experimental and training conditions that guaranteed a very low number of errors in the responses were necessary. The number of subjects used was therefore low and the subjects themselves were very carefully trained. The main results obtained may be summarized as follows: (i) mental rotation requires a period of time that increases linearly with increase in angle size of the transformation to be performed; (ii) rigid and semirigid mental transformations are easy to carry out and RT does not seem to depend on the degree of complexity of the original figure. In spite of this, Cooper and Shepard (1984) report that, when the task is not learnt sufficiently well, there may be an increase in RT as the complexity of the figure increases. They further underline that one of the problems still to be solved concerns the number of details of the corresponding physical objects that mental images maintain when they are transformed.

Our aim in the present work was to study the spatial transformation of figures through mental operations, using a procedure other than the RT method in an attempt to retrieve those immediate and sometimes deviant strategies that were avoided by Shepard's overtrained subjects.

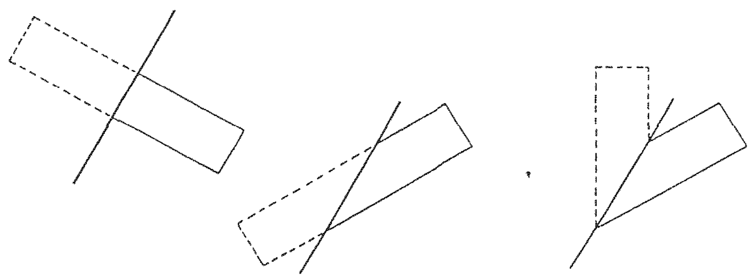
We start from the hypothesis that not all figures, and therefore not all rigid or semi-rigid transformtion tasks, are easy. When conditions are more complex, it should follow that the number of geometrically incorrect responses will increase. The frequency of incorrect responses will be used as a gauge of task difficulty and it becomes important to determine whether incorrect responses are connected to any special strategy used by subjects when facing a given difficulty.

From some preliminary observations (see figures 1a and 1b) in which a quadrilateral had to be rotated round a rotation axis, the task seemed easier and quicker in one condition (1a) than in the other (1b). Furthermore, when considering condition 1b, some subjects gave incorrect solutions, in which the rotated shape was shown with its sides as a continuation of the quadrilateral (figure 2b, dashed part) rather than as the geometrically corrected version (figure 2c, dashed part). Where was the difference between figures 1a and 1b to be found, and therefore the greater difficulty?



(a) (b)

Figure 1. Configurations used in preliminary tests of mental rotation of quadrilateral.



(a) (b) (c)

Figure 2. Responses from preliminary tests. (a) The only result of rotation of shape 1a round the rotation axis. (b) and (c) The two results following rotation of shape 1b round the rotation axis. Dashed line indicates the perceived rotation.

The features of the two configurations must be examined before we can answer this question:
(i) The shape to be rotated is a rectangle in 1a, whereas it is a trapezoid in 1b.
(ii) The quadrilateral axis is orthogonal to the rotation axis in 1a, but not in 1b.
(iii) Once the geometrically correct rotation has been accomplished, the longitudinal sides and the quadrilateral axis simply continue like a mirror image for 1a (see figure 2a) but this is not the case for 1b (see figure 2c).

It could be argued that the direction of the shape to be rotated might be an important piece of information for the execution of this kind of task. It may also be thought that, in accordance with the situations in which the shape is orthogonal to the rotation axis, responses based on the continuation of the original shape would also be made.

Our aim in experiment 1 was to determine which strategies were chosen by subjects when carrying out particular tasks of mental rotation.

2 Experiment 1

2.1 *Method*

2.1.1 *Subjects.* Thirty-six subjects (eighteen males and eighteen females) aged between 19 and 28 years were tested.They were all either graduates or A-level school leavers.

2.1.2 *Stimuli.* The stimuli comprised nine 21 cm × 21 cm white cards. Each card had one thick (0.6 mm) line, 12 cm long, drawn on it. This line, called the rotation axis (RA), could be vertical or rotated clockwise 22° 30′ or 45° from vertical. The centre of this line coincided with the centre of the shorter side of a quadrilateral, 30 mm wide, 60 mm long, line thickness 0.4 mm, measured along its longitudinal axis. Figure 3 illustrates these conditions clearly.

The quadrilateral was always placed to the right of the RA and could have three positions according to the upper angle formed with the RA: 90°, 67° 30′, and 45°. A broken line marked the longitudinal axis (LA) of the quadrilateral. The nine configurations in figure 3 were obtained by combining the three inclinations of the RA and those of the quadrilateral.

In each configuration a semicircle was drawn to the left of the RA. Seventeen dots were placed on this, each marked by a letter from A to S, starting from the top. The gap between one dot and the next subtended angle of 11° 15′ at the centre of the circle. Each stimulus was labelled with two numbers, the first indicated the inclination of the RA to vertical, and the second the inclination of the LA in relation to the RA. For simplicity the stimuli with angles of 67° 30′ and 22° 30′ have been labelled 67 and 22 respectively throughout this paper.

2.1.3 *Procedure.* All subjects were made aware before the test of what mental rotation meant and were shown some real examples of rotations in the form of cardboard cutouts; none of the examples corresponded to the experimental situation. Subjects observed the nine experimental configurations one by one. They were then asked to rotate mentally by 180° the right-hand-side quadrilateral round the RA and then to say on which dot of the semicircle its LA would fall.

Subjects were given the stimuli on a table, at reading distances, in a uniformly lit room. Presentation order was random and different for each subject. No time limit was set for responses.

2.2 Results

In accordance with the hypothesis, the stimuli were subdivided into three groups on the basis of the angle between the position corresponding to the correct response and the position corresponding to the continuation of the LA of the quadrilateral.

The three groups were:
1. stimuli 0/90, 22/90, 45/90, in which the angle taken into account was 0°, since the two positions coincided with dot 'I' in the reference semicircle;
2. stimuli 0/67, 22/67, and 45/67, in which the angle taken into account was 45°, since it fell between dots 'G' (correct) and 'M' (continuation) of the reference semicircle;

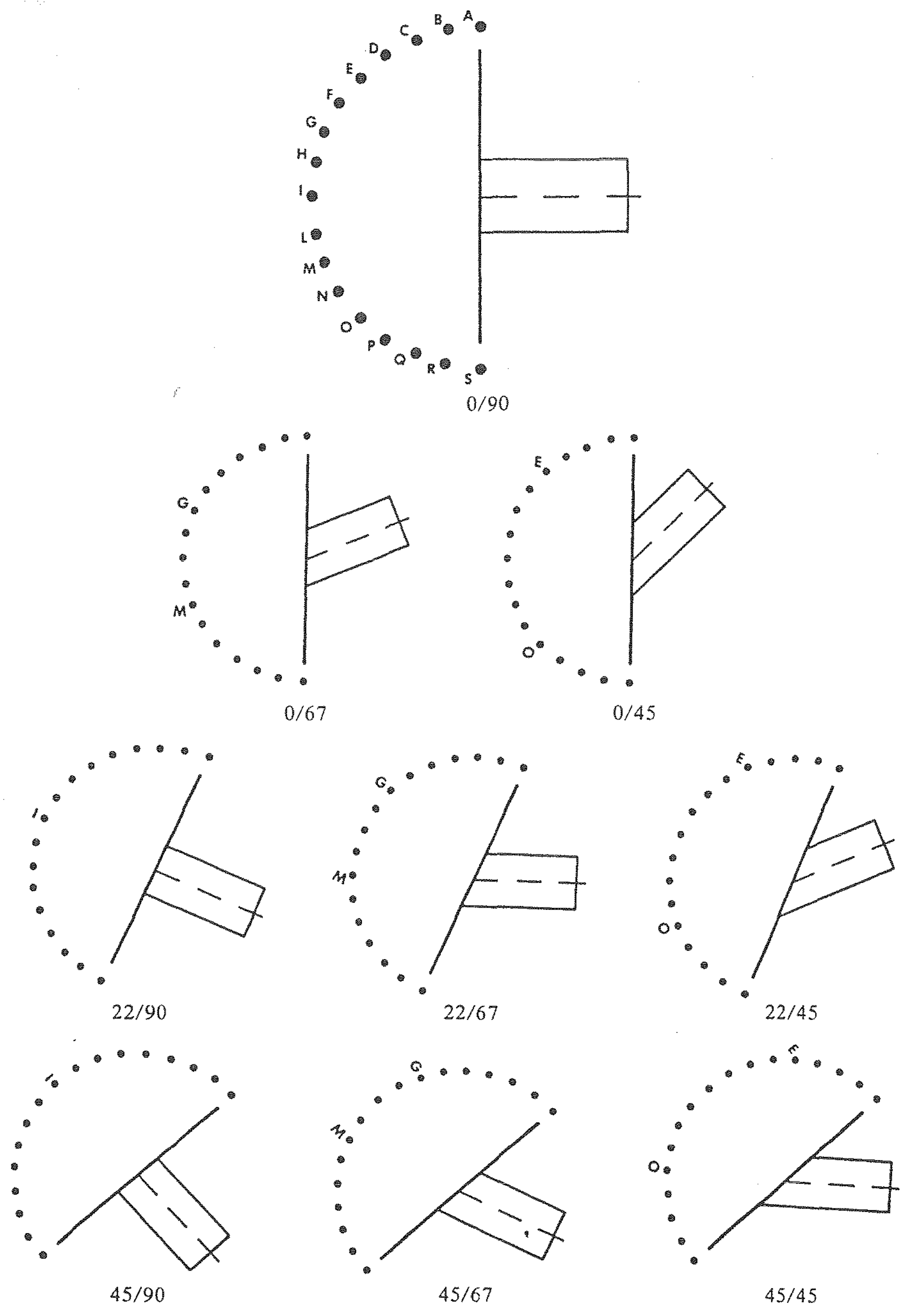


Figure 3. Stimuli used in experiment 1. Each configuration is defined by two numbers: the first indicates the angle of the rotation axis from vertical; the second indicates the angle between the rotation axis and the longitudinal axis of the quadrilateral. The larger (top) figure shows one of the stimuli in its complete form. The smaller figures are simplified representations of the eight stimuli used. Broken line indicates the longitudinal axis of the quadrilateral.

3. stimuli 0/45, 22/45, 45/45, in which the angle was 90°, since it fell between dots 'E' (correct) and 'O' (continuation) of the reference semicircle.

The results show that, in group 1, responses were concentrated on the letter 'I' (correct answer; 93.5 out of 108 or 87%), but, as already said, in this group the correct response and the continuation of the LA of the quadrilateral coincided. In groups 2 and 3, there was a higher total frequency of 'continuation' responses (respectively, letter 'M', 46 out of 108, 43%; frequencies in the other categories never exceeded 14%; and letter 'O', 35.5 out of 108, 33%; frequencies in the other categories never exceeded 13%).

The results obtained for group 1 are so clearly evident as not to require any further analysis; we therefore decided to analyze only the data from groups 2 and 3.

All responses were transformed into degree of shift from the correct response (considered as 0°). This shift was given a positive value if it occurred in the direction of continuation of the LA, and a negative value if it was in the opposite direction. Table 1 shows the mean shift values for each stimulus. The significance of mean shift was calculated using Student's t on a single sample, assuming the value of the correct angle (ie 0°) as the reference mean. As table 1 shows, all shifts were highly significant.

A three-way analysis of variance (ANOVA) was then carried out on these angle data, with sex (two levels) the between-subject variable and RA tilt (two levels, 67° and 45°) and LA tilt (three levels, 0°, 22°, 45°) the within-subject variables. The only significant main effect concerned the RA tilt ($F_{1,34} = 41.81$, $p < 0.0001$). Marginal means turned out to be 27° 43′ for stimuli in which the LA was tilted by 67° with respect to the RA, and 52° 29′ for stimuli in which it was tilted 45° to the RA. It should be stressed that the angle between the position of the correct response and the continuation one was 45° in the former case and 90° in the latter; in both cases the marginal means exceeded half the size of these angles in the direction of continuation. Moreover, the larger these means, the wider the angle between correct and continuation responses in the reference semicircle.

For an overall view of the phenomenon, a different type of analysis was carried out on all response frequencies for the two groups of stimuli. These responses were divided into two types: the geometrically correct responses, and the incorrect (continuation) ones. To define these two areas we used the line bisecting the angle formed by the correct and continuation responses in the reference semicircle. Since the position of the bisecting line always coincided with the position of the letter 'I', we considered as correct those responses falling before 'I', and as continuations those responses falling on or after 'I'. This subdivision provided the data in table 2, where it is shown that the frequency of continuation responses for all stimuli was greater than that of correct responses. Differences also emerge between the various stimuli, as revealed by Cochran's test for comparison between correlated proportions ($Q_5 = 13.21$, $p < 0.05$). The frequency distribution is shown in figure 4.

Table 1. Significance (Student's t) of the mean shift from correct response (0°) in experiment 1.

Stimulus	df	Mean shift	t	p
0/67	35	21° 24′	4.96	0.0001
22/67	35	28° 54′	8.24	0.0001
45/67	35	32° 49′	9.25	0.0001
0/45	35	53° 26′	7.34	0.0001
22/45	35	49° 59′	8.44	0.0001
45/45	35	54° 4′	9.14	0.0001

Table 2. Frequency of correct and incorrect (continuation) responses in experiment 1, grouped according to stimulus and sex.

Response	Stimulus					
	0/67	22/67	45/67	0/45	22/45	45/45
Correct						
male	11	7	3	7	9	9
female	5	6	5	6	7	7
total	16	13	8	13	16	16
Continuation						
male	7	11	15	11	9	9
female	13	12	13	12	11	11
total	20	23	28	23	20	20

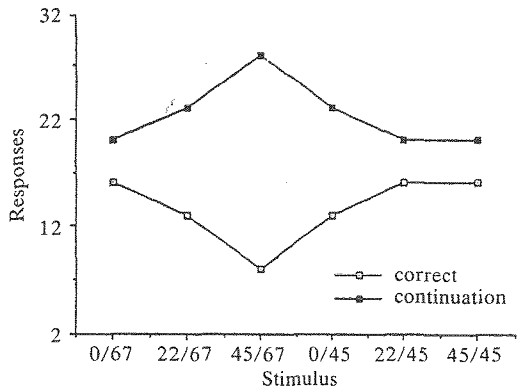


Figure 4. Frequency distribution of correct and incorrect (continuation) responses in experiment 1.

2.3 *Discussion*

From the above results it is clear that there is a fundamental difference between the stimuli in which the quadrilateral LA is orthogonal to the RA (stimuli 0/90, 22/90, 45/90) and those in which there is no orthogonality. The only independent variable used in the first group of stimuli was RA inclination. This did not produce any significant differences in the responses.

In the other two groups of stimuli the independent variables were RA inclination with respect to the vertical and the size of the angle between the LA of the quadrilateral and the RA itself.

Of these two variables, only RA inclination exerted a significant influence on subjects' type of responses. This seems to demonstrate that, when the LA is not orthogonal to the RA, the strategy subjects use to carry out the task mentally is influenced by the shift of the RA from vertical, recognized as one of the main axes of visual space. This shift makes both shape and task more complex and may produce two different performance strategies, according to individual differences among subjects.

One strategy takes into consideration the geometric parameters of the stimuli and arrives at responses that fall in the correct response area. The other stragegy, on the other hand, takes into account the structure and organizational features of the stimuli and arrives at responses that fall in the continuation area.

It may be concluded that, generally speaking, not all rotation tasks have the same degree of difficulty: when the RA moves away from vertical, making the task more complex, two different strategies for task execution emerge, one which we may define as 'organizational-conservative' and the other 'geometric-transformational'. In the first case subjects seem to trust perceptual parameters such as the continuity of direction of the longitudinal sides of the quadrilateral. In the second case, they seem to trust their own inferential abilities to deal with visual information, to the extent that they extrapolate the geometric features of the stimulus.

3 Experiment 2

Experiment 2 was designed to study the situation in which stimulus shape does not end on touching the RA; ie continuity of direction is already present in the starting condition (see figure 5). We used a special kind of mental rotation, termed 'folding'. In this case subjects were presented with straight-sided shapes on which a dotted line appeared (folding line, FL) and asked to hold steady the portion of the shape situated below the FL and to imagine the part of the shape situated above the FL as rotated by 180° around the FL itself: in other words, they were asked to imagine folding the shape along the dotted line.

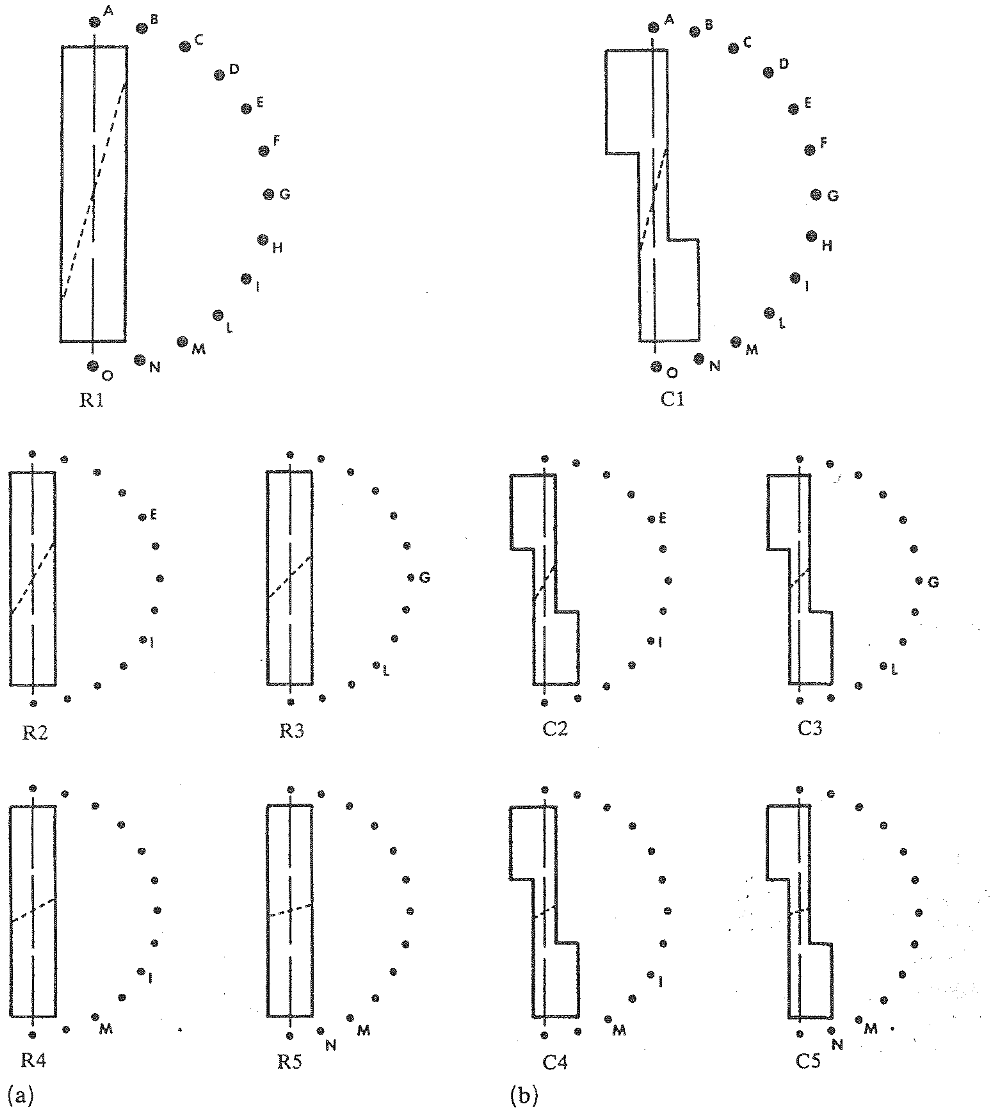


Figure 5. Stimuli used in experiment 2. (a) R series (rectangles), (b) C series (complex shapes). Broken line indicates the longitudinal axis of the rectangle, dotted line the folding line.

It was hypothesized that, in this kind of task, a perceptual-organizational component could be used. This component would be perpendicularity to the FL of the longitudinal axis and the sides of the rotated part of the shape (instead of continuity of the LA and sides as in experiment 1). Two possible types of response were expected: the geometrically correct response, and others which, although geometrically incorrect, would be produced on the basis of the perceptual-organizational factor of orthogonality to the FL. It was also hypothesized that the more complex the shape, the higher would be the number of subjects choosing the second type of response.

3.1 *Method*

3.1.1 *Subjects.* The subjects were twenty-four university students (twelve males and twelve females) from various faculties. They were individually examined.

3.1.2 *Stimuli.* The stimuli comprised two series of five configurations each (see figures 5a and 5b).

(i) R series (rectangle): a 26 mm × 120 mm rectangle with lines 0.6 mm thick was drawn on a 15 cm × 21 cm white card. The longitudinal axis of this rectangle was represented by a broken line 0.4 mm thick. The line was also the diameter of a semicircle drawn to the right of the rectangle and on which thirteen dots marked by thirteen letters of the alphabet were positioned, separated from each other by an equal gap corresponding to an angle of 15° at the centre of the semicircle. The centre of the rectangle was crossed by a dotted line 0.6 mm thick taking five different inclinations (15°, 30°, 45°, 60°, 75°) in relation to the longitudinal axis of the rectangle.

(ii) C series (complex shape): this was made up of five configurations with the same features as those in the R series, except for the fact that the shape was a doubly concave octagon whose longitudinal axis passed through its baricentre (figure 5b).

The ten configurations of the two series were mixed and randomly presented to subjects.

3.1.3 *Procedure.* The stimuli were given to the subjects on a table, at reading distance, in a uniformly lit room. All subjects were made aware of what folding meant and shown some real examples of variously folded cardboard shapes. They were asked to keep steady mentally the portion of the shape placed below the dotted line and to rotate the portion of the shape above it by 180° round the FL until this portion coincided with the portion held steady. In other words, subjects were asked to imagine folding the shape along the dotted line. Once subjects had done this, they were asked to decide on which letter of the reference semicircle the axis of the folded part of the shape would fall. There was no time limit for the execution of this task.

3.2 *Results*

As in experiment 1, subjects' responses were calculated as the angle shifted with respect to the correct response, to which a value of 0° was assigned. For all ten shapes, considered separately, the significance of the shift of the mean of subjects' responses with respect to the correct response was calculated by means of Student's t on a single sample. Table 3 shows that these shifts were highly significant for all shapes except R5 and C5 (in which the FL tilt was 75°).

A three-way ANOVA was carried out on the above data, with sex (two levels) the between-subject variable and type of shape (two levels, R and C), and FL tilt (five levels, 15°, 30°, 45°, 60°, 75°) the within-subject variables. The only significant effect was that of FL tilt ($F_{4,88} = 26.86, p < 0.0001$).

For all the stimuli, we defined the area of correct responses and of incorrect (line orthogonal to the FL) responses. We used the line bisecting the angle between the positions of the letters corresponding to these two kinds of response. This angle continues to decrease as the angle between the FL and the longitudinal axis of the shape approaches orthogonality; in fact, the angle falls between letters 'C' and 'H' (75°) for stimuli R1 and C1; between 'E' and 'I' (60°) for stimuli R2 and C2, between 'G' and 'L' (45°) for stimuli R3 and C3, between 'I' and 'M' (30°) for stimuli R4 and C4, and between 'M' and 'N' (15°) for stimuli R5 and C5 (see figures 5a and 5b).

Table 4 shows the frequencies of all responses, grouped according to stimulus and sex. The bold line divides the correct responses (left), from the orthogonal-to-the-FL responses (right). For both series (R and C) there were fewer correct responses than orthogonal-to-the-FL responses.

Cochran's test was used to carry out an analysis of differences between correlated proportions for the R and C series. Although the difference for FL tilt between the R stimuli ($Q_4 = 11.58, p < 0.025$) was significant, that for the C series was not.

Table 3. Significance (Student's t) of the mean shift from correct response (0°) in experiment 2. R and C denote, respectively, the R and C series of stimuli, with degree of tilt of folding line in relation to stimulus longitudinal axis given.

Stimulus	df	Mean shift	t	p
R1/15°	23	42° 30′	4.79	0.0001
R2/30°	23	48° 45′	7.07	0.0001
R3/45°	23	35° 37′	7.23	0.0001
R4/60°	23	17° 30′	4.90	0.0001
R5/75°	23	3° 12′	0.926	NS
C1/15°	23	53° 7′	6.54	0.0001
C2/30°	23	49° 22′	6.61	0.0001
C3/45°	23	37° 30′	6.65	0.0001
C4/60°	23	19° 22′	4.63	0.0001
C5/75°	23	4° 22′	1.66	NS

Table 4. Frequency of responses for the two series of stimuli in experiment 2, grouped according to stimulus and sex. The bold line divides the correct responses (left) from the incorrect (right; orthogonal-to-the-FL) responses.

Stimulus	Sex	Reference position											
		B	C	D	E	F	G	H	I	L	M	N	O
R1	male		2	4	1	1	2		1		1		
	female	1	4	1				2	2	1	1		
R2	male				2		1	2	3	2	1	1	
	female			1	2	1	1		3	3	1		
R3	male					1	2	1	3	3	1	1	
	female						1	1	3	3	3	1	
R4	male								3	5	1	3	
	female							1	4	2	4	1	
R5	male							1		2	4	5	
	female									3	2	6	1
Total		45 correct										75 incorrect	
C1	male		2	1	2	2	1	2	1			1	
	female	1	1	2			1	4	1	2			
C2	male				2	1	4		3	1			1
	female			1	1	1	1		1	4	2	1	
C3	male						1	4	2	2	2	1	
	female			1			1	1	1	2	4	2	
C4	male							3	2	1	3	2	1
	female								2	4	5	1	
C5	male									4	2	6	
	female									2	3	7	
Total		41 correct										79 incorrect	

The differences among the responses turned out to depend significantly on FL tilt. This result is very probably due to two facts:
(i) Different FL tilts present different degrees of task difficulty; if we go back to table 4, stimuli R3 and C3 (FL tilt 45°) are clearly those which produce the largest number of incorrect (orthogonal-to-the-FL) responses. It must be remembered that the correct response to a 45° tilt is one in which the direction of the rotated portion is orthogonal to the direction of the unfolded portion: this might appear an incongruous and improbable solution to someone performing the operation mentally.
(ii) Since the reference semicircle for responses is fixed with respect to the FL, it follows that with increasing FL tilt the size of the angle between the position of the correct response and that of the orthogonal-to-the-FL response becomes smaller. This reduction should bring about a progressive reduction in incorrect responses. However, the highest number of incorrect responses does not correspond to stimuli R1 and C1 (largest angle), but to R2 and C2, R3 and C3. This leads us to think that it is not the angle, but rather the extent of the modifications that the structural features of the original shape undergo which causes the correct response to be given.

3.3 *Discussion*
The results of experiment 2 confirmed those of experiment 1 as regards the different strategies chosen by individual subjects when performing the task mentally. The perceptual-organizational component used in the processing of the geometrically incorrect responses turns out to be the line orthogonal to the FL, as hypothesized.

The ANOVA showed significance of this factor alone, and not of the complexity of the stimulus due to its features. In the shapes used in experiment 2, the case in which the position of the correct response coincided with that of the orthogonal-to-the-FL response never occurred, as it did for the three stimuli in experiment 1. However, in experiment 2 the shift between the two types of response progressively decreased: for stimuli R5 and C5 this reduction was not significantly far from the geometrically correct response. Interestingly, Cochran's test applied to response frequencies revealed that FL tilt influenced type of response in the case of the R stimuli, but not in that of the C stimuli. This may indicate that the greater complexity of the shape in the latter, although producing a higher number of incorrect responses, is the factor which determines the most frequent type of response. The influence of FL tilt is not seen for the group C stimuli.

4 General discussion
In our opinion, the results of both experiments confirm the following: (i) the fact that—as Perkins (1982, 1983) has convincingly demonstrated in other circumstances—when physical-geometric factors are in conflict with organizational-structural ones, the latter prevail; and (ii) the fact that the fundamental tendency of the perceptual system is to parsimony and invariance, the possible consequence being the sacrifice of the physical-geometric features of stimulus objects.

However, our results also confute the general claim which could be drawn (but which in fact is not drawn by the authors) from the chronometrical investigations on the mental handling of objects by Shepard and co-workers. Particularly relevant to our case is Shepard and Feng's (1972) research on paper folding (reprinted in Shepard and Cooper, 1982, pages 191–206).

According to this claim 'second-order' isomorphism between a perceived object and its mental image can be seen exactly as the correspondence between internal and external states establishing itself during perception. Shepard does not in fact use the term 'identity', but uses instead the terms 'analogy' and 'much in common'. In any case, the fact remains that, in the situations studied by Shepard, a direct relation always exists between reaction time and physical-geometric complexity. Let us now briefly examine this.

A more general consideration persuades us that, in our 'naive' vision of the world, geometric knowledge—in the scientific - mathematical acceptance of the word—finds very little room. In fact, as demonstrated by recent research on 'naive physics' and on the value of mathematical functions, this knowledge, although clearly understood, remains at a declaratory level, very seldom becoming procedural knowledge. In everyday life what matters most is the organizational-figural aspect of objects. The world organizes itself around us, elements segregate themselves and articulate themselves on the basis of the organizational-formative principles we all know. When incongruency exists among the figural features of the stimulation pattern, the perceptual solution that emerges is never dictated by our knowledge of things.

The perceptual system operates according to a 'minimum principle' (Hatfield and Epstein 1985). Its task is to keep the world's objects unchanged. This is particularly evident when complete transformations of stimuli are concerned, as Musatti (1924) was able to demonstrate with stereokinetic phenomena. The mechanisms used to extrapolate these invariants are at the basis of some of the most stimulating recent researches in the psychology of perception, eg the use of group algebra (Hoffman and Dodwell 1985; Palmer 1983).

In complex transformations, saving some invariant features of perceived objects frequently involves the sacrifice of other features. Stereokinetic phenomena demonstrate that object rigidity is one of these: in order to maintain it, tridimensional nonexistent objects are created in the perceptual world. In the cases considered here, it is evident that, because of the impossibility of transforming geometric knowledge into phenomenal experience in imagined situations (ie without constraints derived from external perception), solutions are found which favour the invariance of the main organizational figural elements of the two situations: the continuation of the long sides of the quadrilateral (direction of the shape) in experiment 1, and the orthogonality between the axis of the folded part and the folding line in experiment 2. This sacrifice appears to be conspicuous; nevertheless, it must be observed that, in the first case, the preservation of the direction of the shape after its rotation results in a mirror image which is structurally identical to the first. In the second case, the advantage of orthogonality encourages regularization of the folded border, which tends to approach the shape of a rectangle from its original trapezoid shape (especially in the R series, but generally speaking these remarks also apply to the C series).

We may add that our experiments clearly demonstrate the existence of great differences between the mental rotation of rigid objects and their rotation in the physical world. In our case at least, isomorphism has little meaning. Our experimental conditions, as we have already discussed, are very different from those of Shepard's group. However, it is this kind of research which may provide an answer to the questions asked by Cooper herself (Cooper and Shepard 1984) on the amount and kind of details that are either kept or lost during mental transformation tasks.

Finally, we believe that our results may be usefully related to data from the many studies (Hobbes and Moore 1985; Holland and Quinn 1987) showing how representation of our 'naive' knowledge of the world is shifted with respect to the world as it is described by the natural sciences. This is the field of 'naive physics' (Caramazza et al 1981; Legrenzi 1989; McCloskey and Kohl 1983), but our results may also be extended to the fields of 'naive economics' (Berti and Bombi 1988; Hogarth and Reder 1987) and 'naive mathematics' (Attneave 1962; Luccio 1981).

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