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The ceiling of the Church of St Ignatius and the perception of concave surfaces

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Abstract. This research describes two hitherto unobserved phenomena in the frescoes of the seventeenth century architect and painter, Andrea Pozzo, painted on the vaulted ceiling of the central nave of the Church of St Ignatius in Rome.

The present research also reports the results of two experimental studies on the problem of the perception of shapes projected on concave surfaces. A quantitative evaluation of the phenomena perceived from various points of observation is made by means of stimuli projected at various angles on a semicylindrical surface. The validity of the assumption of invariance, and in particular of the projective invariant called the cross-ratio, is discussed within the framework of ecological theories on perception.

1 Introduction

Between 1685 and 1694 the architect and painter Andrea Pozzo completed his fresco on the ceiling of the Church of St Ignatius in Rome, which depicts "The glory of St. Ignatius as an allegory of the diffusion of the Company of Jesus in the world". Pozzo was able to construct an imaginary extension of the architecture of the church by means of a sophisticated perspective technique on the semicylindrical ceiling. The overall effect is spectacular and so intriguing that an observer standing at the point of projection is, as Pirenne says (1970, page 84), "unable to see the painted surface, *qua* surface. It is impossible to determine where the ceiling surface actually is" (figure 1).

Some psychologists studying perception (Baltrusaitis 1955; Pirenne 1970, 1975; Rock 1984) have been interested in the perceptual phenomena involved in observing the vault of St Ignatius. Of these, Pirenne (1970, pp 79-95) gives a detailed description (partly following Pozzo's book of 1693) of the method used to execute the work, how it appears to an observer, and the perceptual problems it poses to those researching vision.

The present work examined two phenomena which have never previously been described in the literature and which are observed when looking at some details of the St Ignatius fresco from various viewpoints. This phenomenon may be experimentally verified and is closely connected to some perceptual problems recently faced by the ecological approach, such as anamorphoses and perceptual invariants (Johansson et al 1980; Cutting 1986, 1987b; Hagen 1986).

2 Description of the perceptual phenomenon

The Church of St Ignatius has a floor of grey stone with a yellow marble disc inserted into the centre of the nave. This disc indicates the centre of projection, ie the point from which the design of the fresco was projected. If we stand on the disc and look up at the ceiling, we see it as it appears in figure 1. In particular, if we examine the area of ceiling immediately above the main entrance (upper portion of figure 1), we notice that, in the painted architecture, the trabeation on top of the columns and the cornices of the arch appear to be perfectly rectilinear, horizontal, and parallel (figure 2). If, keeping to the centre of the nave, we move from the main entrance to

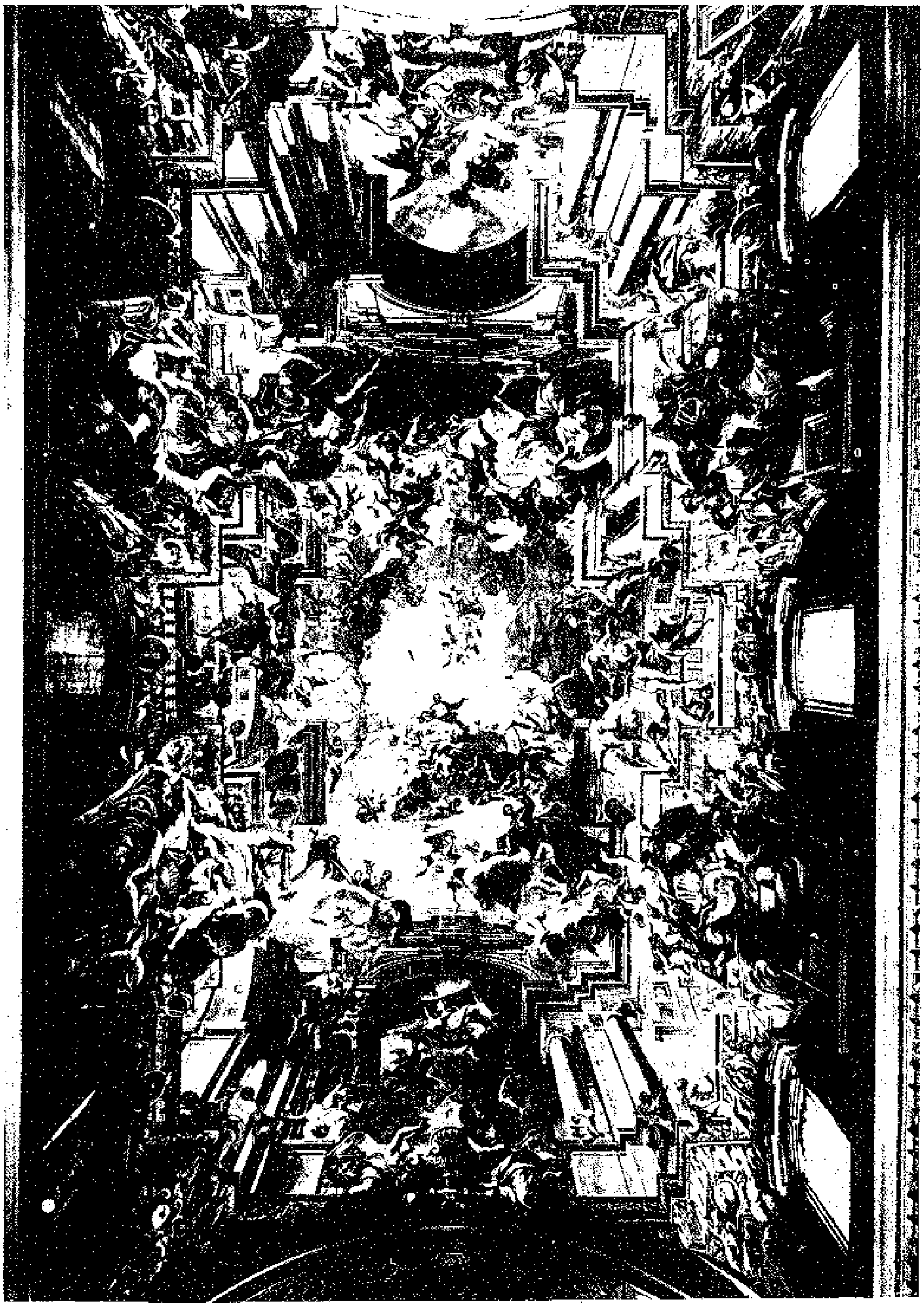

Figure 1. Fresco on the semicylindrical vault of the Church of Si Ignatius in Rome, as viewed from the point of projection (a yellow marble disc in the floor). (Photo: Alinari).

the foot of the main altar, we see that the trabeation and cornices no longer appear to be rectilinear, but curve upwards convexly (figure 3). If we then walk back along the whole length of the church, stand near the entrance again, and look vertically upwards, the trabeation and cornices not only no longer appear to be rectilinear, but this time they curve downwards convexly (figure 4),

This visual experience of a single reality provides three different perceptual results. Moreover, if we ask ourselves which of the three versions corresponds to physical reality, we cannot give a proper answer since, during our visual exploration, our perceptual system has not obtained informtion allowing us to answer this type of question. While observing the ceiling of the church, we undergo an experience which is not frequent in our daily life: observing a portion of reality which is physically rigid and immobile in space but which, if we change our point of observation, also changes, both in form and in position.

Those studying the 'ecological approach' to perception (Gibson 1950, 1979) believe that the 'optical flow' (the ordered and consequent modification of the beam

Figure 2. Fresco on ceiling of St Ignatius: detail over the main entrance, as viewed from the point of projection (yellow marble disc).

of visual angles subtending various points of a scene) conveys all the information necessary for the perception of the rigidity and movement of the objects in that scene. The information used by the perceiver to see the substantially rigid and stable world is composed of the presence of invariants in the optical flow. These invariants are that set of relations and data which remain unchanged during the transformations. In order for the invariants in the optical flow to be perceived, the conditions of observation must be "rich front an ecological point of view". That is, they must not be subject to temporal restrictions (too short presentation) or spatial restrictions (immobility of the observation point). The phenomena which occur during the observation of some of the painted shapes on the St Ignatius ceiling pose two problems to the supporters of the ecological approach to perception.

The first problem is that, although observational conditions may be ecologically rich (the church is well lit and an observer may move about at will), it is impossible to decide exactly whether what we see corresponds to the physical reality of the painting on the ceiling.

The second problem regards the supposed usefulness of the cross-ratio (CR) as a perceptual invariant. The CR mathematically identifies and defines a specific projective property of four aligned dots whose CR remains unaltered, whatever operation of projection or section they undergo (Poncelet 1822; Staudt 1847; Russell 1897).

The geometric significance of the cross-ratio may be summarized as follows: if four aligned points A, B, C, and D are connected by straight lines to the point O, not in line with them, the CR between angular sizes (abed) remains unchanged for all positions of O. Equally, the CR between corresponding linear sizes (ABCD) formed on an arbitrary section plan also remains unchanged.

As well as being a geometric invariant, the CR also seems to be a good candidate as a perceptual invariant in the sense used by Gibson (1966, page 145) as "a kind of stimulus information for the constant properties of the object". Gibson et al (1978)

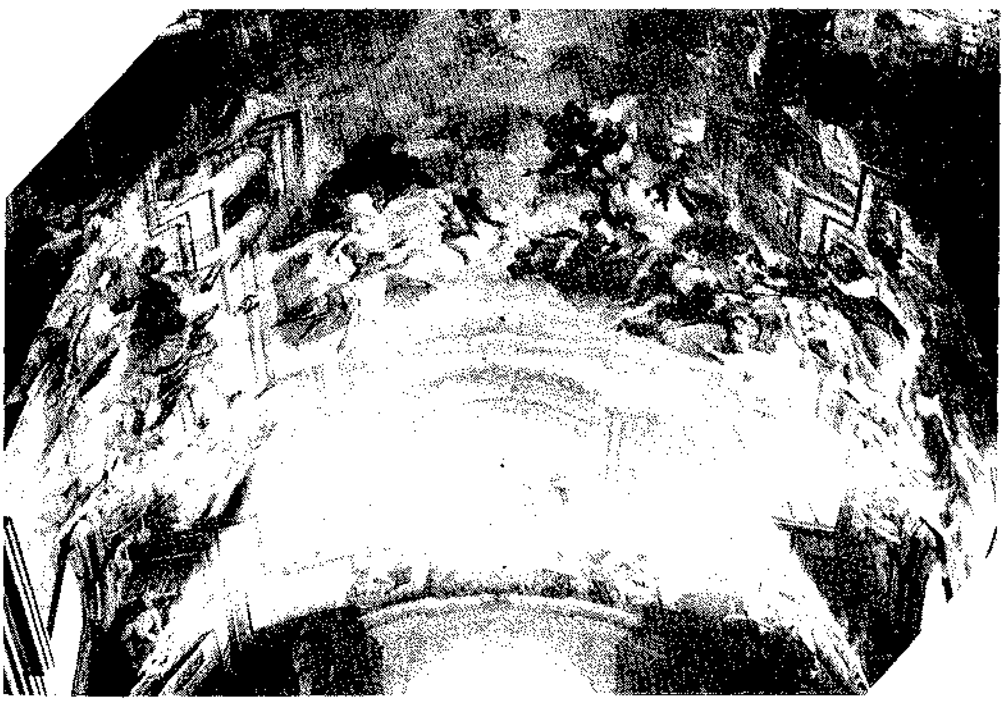

Figure 3. Same detail as in figure 2, as seen from opposite end of nave, at the foot of the main altar.

and Shaw and Pittenger (1977) hypothesized that the CR was a good perceptual invariant, and although their hypothesis was taken up by and shared by other researchers (Johansson et al 1980; Cutting 1986, 1987a; Hagen 1986), it was questioned by others (Epstein and Park 1986; Simpson 1986; Massironi and Savardi 1987, 1991; Niall and Macnamara 1989).

As far as we are concerned, it must be remembered that, in projection, CR invariance has been demonstrated even when the section of the rays converging on the four aligned dots is produced by a second-order conical surface (Staudt 1847)—as is the case for the semicylindrical vault of the Church of St Ignatius.

As we have noted, the phenomenal experience of observing the church ceiling is that of a lack of rigidity in the architecture depicted. The horizontal structures sometimes appear to be rectilinear and sometimes curved and, in the latter case, curved upwards or downwards according to the point of observation. Since the CR among any four points of the structure remains constant, it must be deduced that, at least in this case, the CR, being a projective invariant, cannot also be a perceptual invariant. These brief remarks also show that the phenomenon deserves more

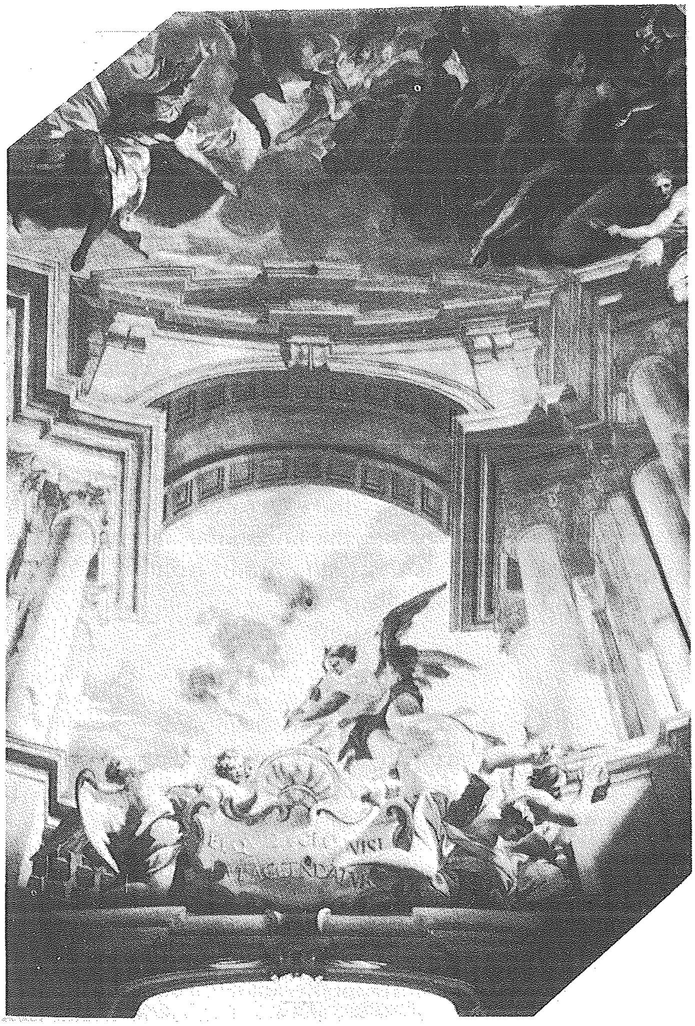

Figure 4. Same detail as in figure 2, as seen from near the main entrance.

accurate experimental study, as noted by Cutting (1987a, page 74), who emphasized the necessity to conduct research-on limit conditions and on the passage from rigidity to nonrigidity. Other authors too (Jerison 1967; Deregowski and Parker 1988), by using a different approach, have experimented with the problem of the effect of line of sight on the perception of spatial configurations painted on a two-dimensional surface. In particular, the latter authors used a painting by Vermeer ("The Music Lesson"), which shows a concave trihedron formed by the perpendicular meeting of two walls and the floor of a room. One of the walls is parallel to the surface of the painting and the angle formed by one wall and the floor is judged to change inclination according to the position of an observer moving along a line parallel to the plane of the painting. In order to explain this result, the authors hypothesized "the presence of specific standard references in architectural painting which permit the viewer to register the perceptual changes easily" (page 20). This is in agreement with the distinction, proposed by Deregowski (1980, 1983) and taken up again by Deregowski and Parker (1988), "between those pictures which represent but which do not evoke the illusion of depth (called epitomic) and those which convey pictorial depth without necessarily representing any known object (called eidolic)" (page 20).

The phenomenon observed in the Church of St Ignatius is another interesting condition allowing the study of perceptual changes deriving from observer movement.

Shapes painted on a concave surface produce greater and more evident effects than those painted on flat surfaces on which any change in viewpoint may, in suitable conditions, cause a phenomenal change in the position in space of the observed subject, but no elastic change in its shape. We therefore decided to carry out two experiments with specially prepared experimental apparatus which reproduced the perceptual phenomena observed in St Ignatius in a controlled way, in order (i) to verify if the observed phenomena also occurs in a much more simplified situation with nonfigurative stimuli; (ii) to measure the extent of the phenomena, by means of a specially designed quantitative criterion; and (iii) to identify and analyze the causes of the phenomena.

3 Apparatus

A semicylindrical vault, painted matt black, 2.10 m long, with an internal diameter of 82 cm, and a maximum internal height from the ground of 4.05 m, was placed in a corridor 10.40 m long and 1 m wide. Figure 5 shows the exact position of this vault in the corridor. A slide projector, tilted at 45° from the horizontal and with a lens height of 1.87 m, was placed on the same axis as the vault, 1 m in front of it. In this way, the images of projected slides met the vault slightly beyond its centre.

4 Experiment 1

4.1 Subjects

Our subjects were ten students, either about to graduate from the University of Padova or carrying out research in the Department of Psychology of this university. They had all followed courses and taken examinations in the psychology of perception. All subjects had normal vision and none wore glasses.

4.2 Stimuli

Three stimulus slides were used:

(i) a continuous-line stimulus consisting of a rectilinear, horizontal segment, 20 mm long and 0.4 mm thick on the slide;

(ii) an aligned-dots stimulus consisting of four horizontally-aligned, equidistant dots, the overall length of which, from the extreme left of the first dot to the extreme right of the fourth dot, was 20 mm on the slide, and dot diameter was 0.6 mm;

(iii) a circle stimulus consisting of circle with a slide diameter of 13 mm and having a line thickness of 0.4 mm.

All three stimuli were white on a black background, so that no background appeared when they were projected on the vault.

4.3 Procedure

The experimental corridor was weakly illuminated, but in such a way that the contours of the vault and other objects (projector, cupboards, doors) were clearly visible. The experiment was subdivided into two sessions: the first was to familiarize subjects with the problem by means of a description of the phenomenon; the second was to ask them to give quantitative judgments about the effects they observed.

4.3.1 *Session 1: description of the phenomena*

Subjects spent about 5 min in the corridor to accustom their eyes to the semidarkness. Then one of the three stimuli was projected onto the vault, and the experimenter asked subjects to move freely along the whole length of the corridor, watching the projected shape and describing all changes noted during this exploration.

All subjects noted the following modifications in the stimuli:

Continuous-line (CL) stimulus. At a distance of 3.80 m behind the projector (henceforth, for the sake of simplicity, called +400), subjects saw a concave-upward arch, not resting on the vault but suspended and slightly tilted under it. At about 2 m behind the projector (position +200) the arch seemed less curved, but lower with respect to the top of the vault.

When level with the projector (position 0), subjects saw a straight line definitely not touching the vault and seeming to end in two points symmetrically opposite with respect to the longitudinal plane of the vault (ie it appeared like a straight neon tube, transversally bisecting the vault but not resting on its surface).

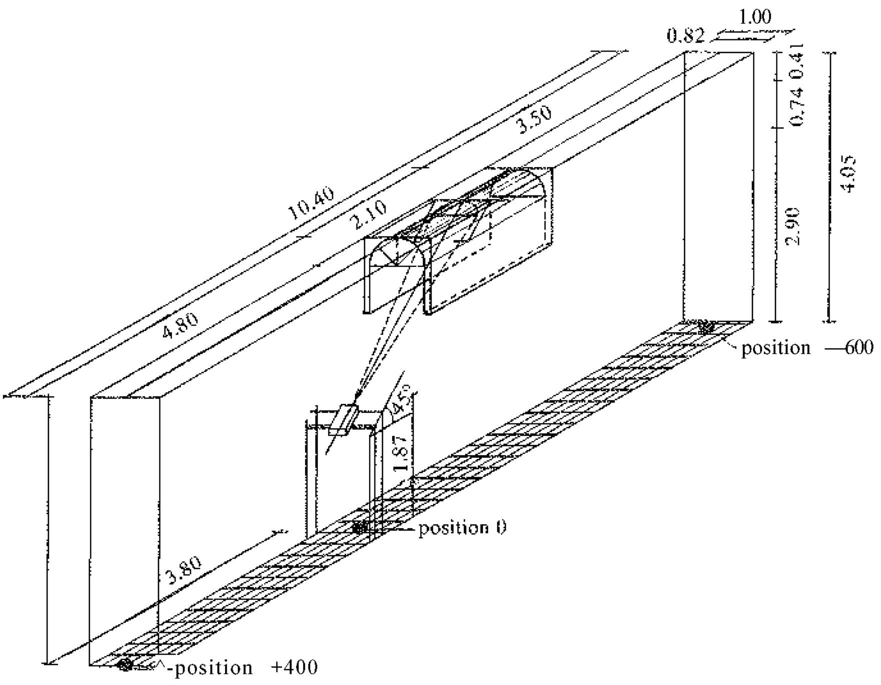

Figure 5. Schematic representation of the experimental apparatus, showing the values of various distances (see text). Three of the positions from which subjects had to make evaluations during the experiments are shown as black circles.

At about 2 m in front of the projector (position, -200), the straight line was transformed into a concave-downward arch, the vertex of which almost touched the top of the vault. At about 6 m in front of the projector (position -600), the arch became much more curved and acutely angled, with its vertex touching the top of the vault and its ends slightly detached from the vault and pointing downwards.

Aligned-dots (AD) stimulus. Perceptual experience was similar to that described for the CL stimulus, except that the AD stimulus seemed to be less intense and, above all, the dots appeared to be nearer to the surface of the vault.

Circle stimulus. At position +400, subjects saw a horizontal, elliptical, flattened shape, suspended inside the vault, practically without contact with it. From +400 to +200 and as far as position 0, the ellipse became more and more circular until, at position 0, it appeared as a perfect circle suspended in the vault, but not touching it.

From position 0 to -200, this circle was transformed into a slightly oval vertical ellipse. At position -600 it became an ovoid, wide at the base and much narrower at the top, and tilted in space with its narrowest part almost touching the top of the vault.

4.3.2 *Session 2*

After session 1, subjects were asked to estimate some parameters of the three stimuli from each of the five observation distances (+400, +200, 0, -200, and -600) for the CL and AD stimuli in the following way.

The unit of measurement that subjects had to use was the overall length of the stimulus observed from position 0 (thus seen as a straight line), to which the value of 10 was assigned. Subjects used this value as a standard to evaluate, for each of the five positions and each of the two stimuli, the following parameters (shown in figure 6): *X,* the vertical distance from the centre of the stimulus to the top of the vault; *Y,* the vertical distance from the ends of the stimulus to the top of the vault; Z, the horizontal distance from the ends of the stimulus to the sides of the vault; and *a/b,* the ratio between the vertical segment joining the vertex of the arch with the chord subtending it and the chord itself.

For the circle stimulus, subjects were asked to evaluate parameters *X, Y,* and Z for the horizontal diameter, while *a/b* was the ratio between the horizontal and the vertical diameters. The unit of measurement was the horizontal diameter observed from position 0, to which a conventional measure of 10 was assigned.

Figure 7 shows three photographs of the experimental apparatus when the CL stimulus was projected on the vault. These photographs were taken from the three observation positions, +400, 0, and -600, respectively.

The experiment lasted for about 1 h per subject. While all subjects found that qualitative transformations were very easy to see and describe, quantitative judgments were more difficult and required more time to make.

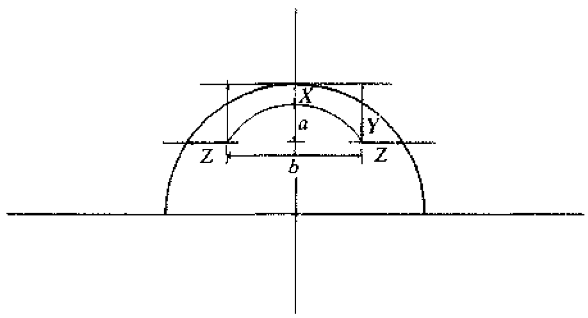

Figure 6. Parameters which subjects were asked to evaluate during the experiment (see text).

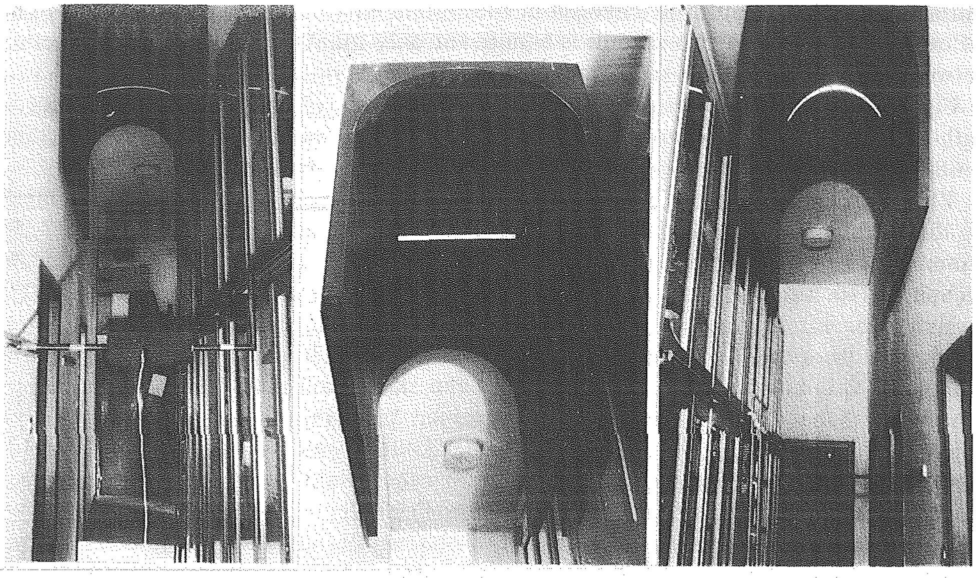

Figure 7. Photographs showing the experimental apparatus when the continuous-line stimulus was projected onto the vault. The photographs were taken from (left) position +400, (centre) position 0, and (right) position −600.

4.4 Results

The data collected were subjected to a two-way ANOVA for repeated measures (distance: 5 levels; stimuli: 2 levels). It was thus possible to verify, on the one hand, the differences in the various parameters when the point of observation was changed and, on the other, whether the CL and AD stimuli were seen to become deformed in the same way or not. The ANOVA was carried out separately for the four parameters X, Y, Z, and a/b. The results (table 1) show that a change in the observation distance

Table 1. The results of a two-way ANOVA for repeated measures for the parameters X, Y, Z, and a/b, as estimated in experiment 1.

Parameter	Distance		Stimulus		Interaction	
	$F_{4,90}$	p	$F_{1,90}$	p	$F_{4,90}$	p
X	126.5	<0.0001	7.72	<0.01	4.63	<0.005
Y	10.64	<0.0001	0.12	>0.25*	1.05	>0.25*
Z	16.12	<0.0001	2.79	>0.05*	7.35	<0.001
a/b	61.05	<0.0001	17.03	<0.005	2.36	>0.05*

* not significant

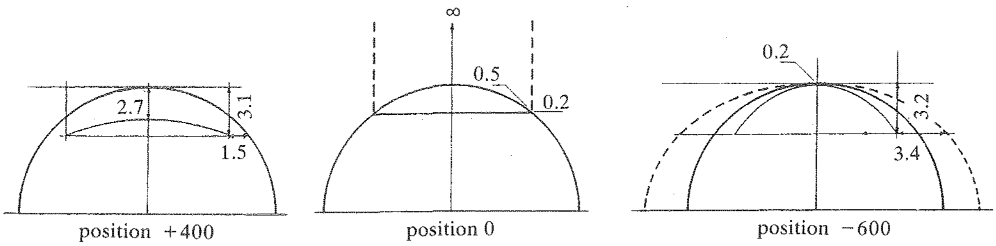

Figure 8. Schematic representations of the phenomenal shape and position of the continuous-line stimulus when observed from positions of +400, 0, and −600. The data correspond to means of subjects' responses.

causes statistically significant changes in the estimated values of all parameters. The difference between the two stimuli is significant only for X and a/b. The interaction between the two variables is significant only for X and Z.

Figure 8 shows the positions that the various points of the CL stimulus take on with respect to the surface of the vault, when observed from positions +400, 0, and −600. The curves are derived from the mean values of the parameters measured.

The values shown in the three representations of figure 8 correspond to the mean of the estimates for X, Y, and Z in figure 6. The schematic representation of the stimulus observed from position 0 gives the value of X as infinite, due to the subjects' incapacity to judge the distance from the centre of the vault when they saw the straight line cut it without resting on it. In effect, when this perceptual result was produced, the sense of the curve of the vault was lost—as happens when observing the ceiling of St Ignatius (figure 1). As regards the sketch for position −600 (figure 8), when the shape of the stimulus curve, obtained from the values of a/b, had been calculated, Z was found to be greater than the distance which is in fact possible inside our semicylinder. Subjects' comments thus revealed that, from position −600, they saw the vault as wider than it actually was (dotted line in figure 8).

As regards data on the circle stimulus, a one-way ANOVA for repeated measures (distance: 5 levels) was carried out for each of the four parameters (parameter X: $F_{9,40} = 4.47$, $p < 0.005$; parameter Y: $F_{9,40} = 8.37$, $p < 0.005$; parameter Z: $F_{9,40} = 6.98$, $p < 0.005$; parameter a/b: $F_{9,40} = 24.26$, $p < 0.005$). These results clearly show that, when the observation point is changed, both the shape of the stimulus and its position with respect to the surface of the vault also appear to change.

4.5 Discussion

In table 1, the data in the Distance column clearly show that, according to the position from which a stimulus projected onto a concave surface is observed, the considered parameters (see figure 6) change. This means that both the perceived shape and the position of the observed configuration change, and thus that the configuration itself does not remain phenomenally rigid. This conclusion gives rise to two theoretical consequences, briefly outlined below:

(i) In a situation such as the one considered here, processes of perceptual constancy do not occur, or at least they do not occur in an evident way. This aspect will be amplified in section 6.

(ii) The observed transformations occur in conditions of perfect perspectivity, in which the cross-ratio therefore remains unvaried. In this case, as some researchers have thought, the CR does not seem to provide useful information on the perception of rigidity. In this context, Cutting (1986, page 70) states: "At best, invariants work for perceivers only some of the time, and the determination of how well they work for psychologists is an empirical question." In the light of this statement, we can conclude for the time being that the condition we studied is a case in which the CR, although working well as a projective invariant, works very badly as a perceptual invariant.

Let us now briefly consider the results of our statistical analyses of the data. The Stimulus column in table 1 shows that there is a statistically significant difference between the CL and AD stimuli only for parameters X and a/b. In the graph shown in figure 9a, which was drawn from the mean values of parameter X, we see that the difference between the CL and AD stimuli is only valid at position +200. These values mean that, at that distance, the dots appear nearer the top of the vault with respect to the line. In order to understand this result, it must be recalled that the dots, being four in number and regularly spaced, leave an empty space between the two inner dots, and it was precisely in that space that the value of X was to be judged, so that the CL stimulus which, in position 0 appeared to be completely independent of the vault, appeared in position +200 to be even farther from its surface with respect to the dots.

Figure 9b shows the trend of the values of parameter a/b for the CL and AD stimuli. Except for at positions 0 and −600, the CL stimulus again shows higher values than those assigned to the AD stimulus. That is, the virtual line joining the dots curves less than the continuous line. The separation between the various parts of the AD stimulus probably means that they appear less deformed than a strongly unitary structure like a line. Here, we could also invoke the computability of the CR which would be possible only in the case of the four aligned dots and not in that of the continuous line; this computability would give better information, leading to less deformation. However, in our opinion, this is not the reason for the greater apparent rigidity of the dots with respect to the line, bearing in mind that the dots too, although appearing less deformed, do not appear to be rigid.

The third column of table 1, Interaction, reveals a significant interaction between the type of stimulus and the distance with respect to parameters X and Z. Parameter X is a measure of the distance from the centre of the stimulus to the top of the vault; Z is a measure of the distance from the ends of the stimulus to the vault. The fact that this interaction emerges shows that, when the observer is far from the point of projection (position −600), the ends of the continuous line appear to be farther horizontally from the surface of the vault, than do the dots (figure 9c). When the stimuli are observed from position −200, distance Z appears to be the same for the dots and the line. When an observer stands at position 0 or +200, distance Z appears greater for the AD stimulus than for the CL stimulus (figure 9c). Lastly, from position +400, distance Z appears greater for the CL stimulus than for the AD stimulus. This means that configurations drawn on a semicylindrical vault appear to change in different ways from various viewpoints, according to their structure. These differences appear to depend on the fact that the CL stimulus is unitary and continuous, whereas the AD stimulus is made up of four separate elements.

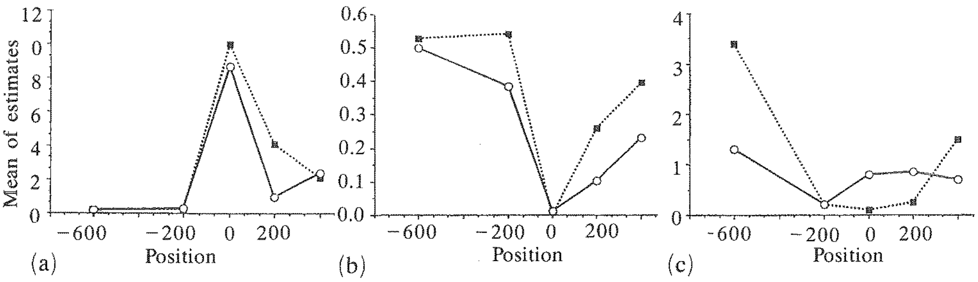

Figure 9. Mean values for estimates of (a) parameter X, (b) parameter a/b, and (c) parameter Z for the CL stimulus (filled squares) and for the AD stimulus (open circles) from different positions in experiment 1.

5 Experiment 2

In experiment 1 the stimuli were projected onto the semicylindrical vault at 45° from the horizontal. During our observations of the ceiling of St Ignatius, we noticed that when the observation point was changed, the configurations at the edges seemed to change much more obviously than those in the centre. However, if we project a two-dimensional drawing onto a semicylindrical ceiling from a central point (the yellow marble disc on the floor), as Pozzo hmself (1693) says he did, the projected rays strike the vault above the yellow disc at 90°, while they reach the farther edges of the vault at a much smaller angle. In our opinion, smaller phenomenal changes should occur in the first case than in the second. In order to verify this hypothesis, we decided to show the same stimuli, this time projecting them at 90° from the horizontal (instead of along a plane tilted at 45°, as in experiment 1).

5.1 Subjects and procedure

There were five subjects, all studying psychology at the University of Padova, who had followed courses and taken examinations on the psychyology of perception.

The procedure and instructions were the same as in experiment 1 but with the following two differences. (a) As mentioned above, instead of being tilted at 45° from the horizontal, the projector was set at 90° from the horizontal, so that the projection plane cut the vault along a perfect semicircle (and not along an elliptical arc as in experiment 1). (b) Only two stimuli were used, ie the continuous line (CL) and the four aligned dots (AD), because our subjects had found that the parameters were easier to see and evaluate for these stimuli than for the circle stimulus. Since when subjects moved away from the projector in either of the two opposite directions, the effect on the observed stimuli appeared to be the same, we decided to request only three judgments per stimulus, at positions +400, +200, and 0. The parameters were the same, ie X, Y, Z, and a/b (figure 6).

5.2 Results and discussion

The data collected allowed us to carry out a two-way ANOVA for repeated measures (distance: 3 levels; stimuli: 2 levels) for each parameter. Results are shown in table 2. The following observations can be made:

(i) Measurements of parameter Z were not analyzed because all responses were zero. This result means that, for all subjects, the ends of the stimulus touched the side of the vault for all observation distances. This is also confirmed by the nonsignificance which emerged from comparisons with parameter Y.

(ii) A significant distance effect emerged regarding the distance factor for parameter X. This means that, when the observation point is changed, the distance between the centre of the stimulus and the top of the vault also changes.

(iii) Significant differences were found between the values of parameter a/b, for both factors (distance and stimulus) and for their interaction. This result shows that phenomenally obvious changes take place when the observation point is changed, even when the stimulus is projected at right angles to the top of the vault. However, when the angle between the projection axis and the top of the vault was not 90°, these changes were less intense than those found in experiment 1. The changes particularly concern the ratio a/b (figure 6) (ie the degree of apparent curvature of the stimulus), and the tips of both stimuli never appear to be detached from the surface of the vault. These results indicate that the extent of this curvature depends on the fact that the stimulus was a unitary shape (a line), not a composite shape (four aligned dots); the latter appear to change less than the former (figure 10). It should be recalled that in experiment 1 the distal stimulus was an elliptical arc, while in experiment 2 it was a semicircle. This aspect of regularity may also be presumed to play an important role in keeping perception more constant.

Table 2. The results of a two-way ANOVA for repeated measures for the parameters X, Y, Y, and a/b, as estimated in experiment 2.

Parameter	Distance		Stimulus		Interaction	
	$F_{2,24}$	p	$F_{1,24}$	p	$F_{2,24}$	p
X	47.39	<0.0001	1.02	>0.25*	3.07	>0.05*
Y	2.67	>0.05*	0.73	>0.25*	1.44	>0.25*
a/b	102.5	<0.0001	18.26	<0.005	7.39	<0.005

* not significant

Interpreting the results of both experiments from the viewpoint of the rigidity of the observed object, it may be said that in both cases the stimuli are not seen as rigid when the observation point is changed, but that nonrigid transformations are more intense when the stimuli are projected at an angle of 45° from the horizontal, than when they are projected at 90° from the horizontal.

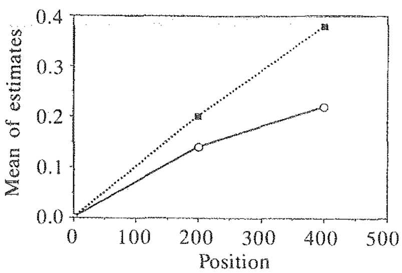

Figure 10. Mean values for estimates of parameter a/b for the CL stimulus (filled squares) and for the AD stimulus (open circles) from different positions in experiment 2.

6 Conclusions

In order to attempt an explanation of the phenomenon demonstrated in our two experiments, it should be recalled that changing the observation point always causes projective transformations of a rigid stationary stimulus although, geometrically specified, these are not always and not necessarily reflected in perceptual results. It may be said that, in any process which implies spatial depth, mainly optic and/or mainly psychological aspects may be identified. The former are those in which the perceptual result reflects retinal recording (eg two stimuli are recorded as one large and one small, and are perceived as one large and one small). The latter are those in which there is no close correspondence between retinal recording and the perceptual result (eg two stimuli are recorded as one large and one small, but are perceived as having the same size and being at different distances from the observer). The so-called constancy phenomena are typical psychological phenomena in that optical modifications of shape and size are perceived as variations in distance or in degree of tilting of objects with constant shape and/or size.

The problem becomes more complicated when passing from the perception of three-dimensional objects observed in space to perception of objects represented on a two-dimensional surface, such as paintings or photographs (Hagen 1980). The phenomenon is fully presented when a painting or photograph is observed from a point different from that used by the painter to create his picture or by the photographer to take his photograph. In this context, Rosinski and Farber (1980, page 138) observe that:

> Because the precise relationship between viewing point, dislocations, geometric transformations, and perceptual accuracy is unknown, determination of the correct viewing point for display and of the effects of displacement is crucial for a further understanding of picture perception.

The studies to which the above authors refer deal with pictures painted on flat surfaces. The ceiling of St Ignatius is a singular case of pictorial representation, since its surface is not flat but semicylindrical. In our experiments the figures projected on the vault constituted the pictorial representation, and modifications perceived by the observers at the various observation points showed both an optical-geometric component and a psychological component. Let us try to separate these two aspects.

We must first consider what geometric transformations the stimuli undergo when the observation point is changed. Figure 11a is an axonometric representation, showing the situations in which continuous-line stimulus a is viewed from distance 0 (projector tilted at 45°) and observation is made from positions 0 and +400. Segment a indicates the stimulus on the slide to be projected, and its projection on the vault is obtained by means of parallel, not converging, rays (as occurs in reality with a slide projector). This simplification is to facilitate the comprehension of the following reasoning.

Plane α is formed when the stimulus is projected and cuts the vault along the elliptical arc a. If this arc is observed from position 0, it is seen as the straight line b, cutting the vault.

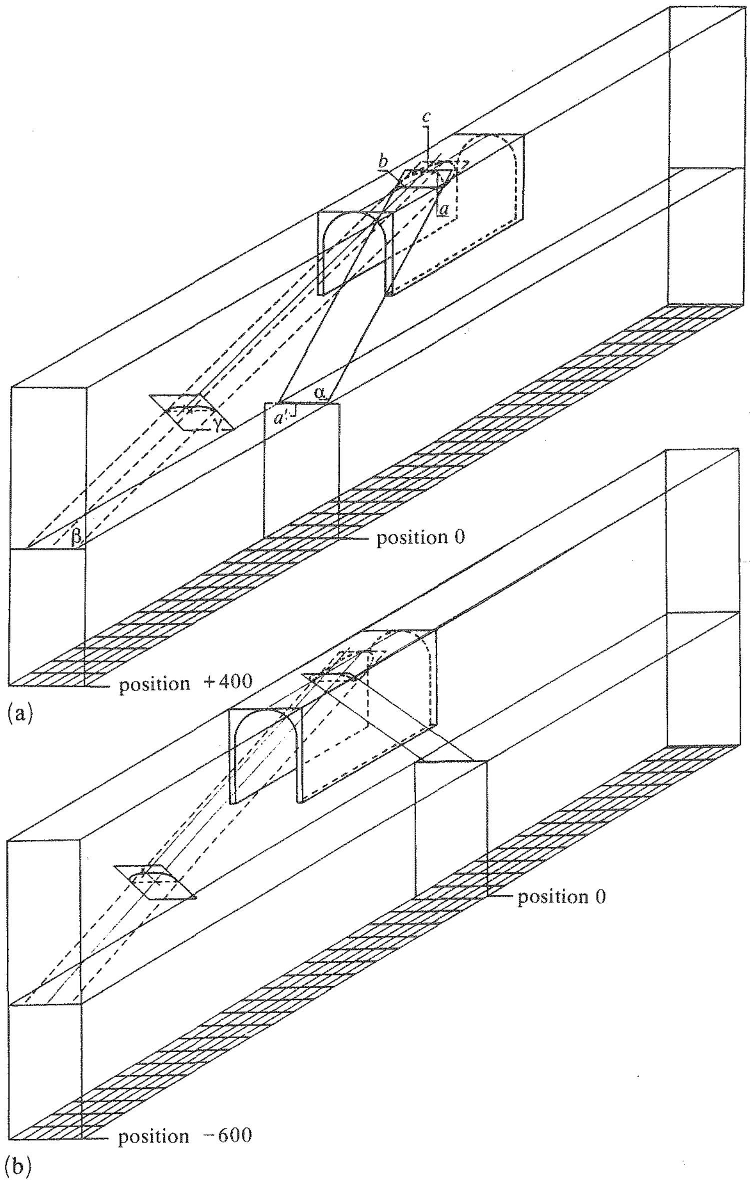

Figure 11. Schematic representation of the geometric changes in the continuous-line stimulus observed from (a) positions 0 and +400 and (b) position −600 (see text for detailed explanation).

In order to understand what happens in the projection when the same stimulus is viewed from position +400, we must reason briefly ab absurdo. To be able to see segment b as a straight line from position +400, the stimulus should coincide with the elliptical arc c, which is the line along which plane β (the plane on which both the observer's eye and segment b lie) meets the surface of the vault. However, since the projector has not been moved and the projected image is always the elliptical arc a, the new configuration will be halfway between arc a and the straight line b. For the sake of simplicity, let us observe what happens on the plane orthogonal to plane β (plane γ may be understood as a metaphor for the retina, the visual plane, or for Alberti's window). On this plane, the projected vertex of arc a falls higher than that of the elliptical arc c, which makes the line seem to curve upwards.

A similar situation, although in the opposite direction, occurs when the observation point is at position −600 (see figure 11b).

The perceived deformations seem to have the same characteristics as geometric-projective deformations produced on a two-dimensional plane at varying observation distances. This evidence seems to support the intuition of the Renaissance perspective painters who hypothesized the existence of a two-dimensional plane—the well-known Alberti's window—which cuts the beam of rays converging in the observer's eye and on which a two-dimensional projection of the three-dimensional world was drawn. Gibson (1950) also took up this hypothesis, and Hagen (1980) uses "Alberti's window" as the subtitle for the first of two volumes she has edited on pictorial perception. But we still cannot decide whether this plane is a purely cognitive construct, a sort of mental plane, or—much more simply—the retina. Available data seem to indicate that the perceptual process uses a two-dimensional visual plane on which the optical array or optical flow transcribe projective information which are usually sufficient to account for the rigidity and position of objects in space.

If we hypothesize that our perceptual system uses as cognitive apparatus a sort of 'internal Alberti's window'—that is, a two-dimensional plane, always at right angles to the optical axis, on which the observed stimuli are projected—most of the problems emerging from the observation of two-dimensional pictorial representations may be thought to derive from the degree of correspondence which is created between the 'internal projective plane' (that of the observer) and what we may define as the 'external projective plane' (that of the painting). When the two planes are parallel and the viewing point of the former corresponds to that of the latter, there is perfect correspondence between the two, and there are practically no differences, on a perceptual level, between information coming from the external world and that coming from its pictorial representation. If the two planes remain parallel, but there is no coincidence between the viewing points, small perceptual distortions are produced regarding the phenomenic distance of the objects depicted [cf the examples reported by Rosinski and Farber (1980)]. When there is neither parallelism between planes nor coincidence between viewing points, perceptual distortions may arise regarding the position of objects—the farther away the two viewing points, the greater the distortions (Jerison 1967; Deregowski and Parker 1988). In these cases, it seems that the perceiver cannot take into account two discordant viewing points—that of actual observation and that used in the construction of the picture—at the same time. Moreover, a specialized system for collecting information regarding the arrangement of objects in three-dimensional space need not necessarily be specialized in order to bear in mind the exact characteristics of a virtual space represented on an external surface. Deregowski and Parker (1988) reported that their subjects gave completely correct judgments when, instead of being presented with a pictorial image or a graphic simplification of the stimulus (a dihedron), they observed a three-dimensional structure made of wire which exactly reproduced the angles of the stimulus.

These results seem indirectly to confirm the hypothesis of the perceptual use of a kind of internal projective plane. Another indirect confirmation may be found in our results in which the discrepancy between internal and external projective planes no longer deals exclusively with their reciprocal degree of tilt (as the cases in the literature show), but also with their degree of curvature, since the former seems to be always and only flat, while the latter may take on various degrees of curvature. In our case, this degree of curvature was semicylindrical.

We have seen that the shape of the perceived stimulus corresponds to the optical-geometric projection of a figure drawn on a two-dimensional plane on a semicylindrical vault. Figures 7, 8, and 9 show that the stimulus is also seen to change its position in space according to where it is viewed from. This result must be considered exquisitely psychological, since it depends mainly on the perceptual processes activated. Note that, although the stimuli always lie on the surface of the vault, they are seen as being suspended in various ways inside it. The maximum degree of independence of the stimulus from the surface on which it is projected occurs when the observer is at position 0, as happens when looking at the ceiling of St Ignatius from the regularization point.

In the classic constancy processes, it may happen that observer or object movements produce shape or size changes which are not perceived as such. This is because changes in observer or object position are perceived, while changes in the shape or size of the object are not, so that the latter seems to remain unchanged.

The cases we examined concern two phenomena which are usually independent but occur at the same time: (i) Optical-geometric changes in stimulus shape are seen as they are produced projectively, and we have the perception of nonrigid forms which change shape according to the point from which they are observed. (ii) Changes in stimulus position are also seen when the observation point is changed and the stimulus appears to hang in space, detached from the surface on which it is projected. All this occurs in spite of Gibson's hypothesis and the ecological hypothesis on the use of projective invariants as rigidity information; these hypotheses do not seem to apply in this case. Our experimental conditions involved complete and perfect perspective and thus guaranteed the preservation of that special projective invariant which is the cross-ratio. However, when faced with a semicylindrical surface, this invariant ensures neither preservation of rigidity of the observed object nor its precise location in space. This also occurs both in environmental conditions and in observational situations which are sufficiently rich from the ecological viewpoint. Cutting (1986, pages 137 and 142) believes that the CR is perceptually important only when a planar object rotates. If this is true, then we cannot expect our configurations (which are not planar) to remain rigid, even though the projective invariants were preserved.

In brief, in our experimental conditions the normal constancy processes on which perceptual activity is based no longer seem to function. We must then ask what mechanisms and laws regulate the perception of configurations drawn on concave surfaces. For the time being, our ideas are hazy, and the hypothesis of a kind of 'internal Alberti's window', flat and at right angles to the optical plane and matched with pictorial surfaces of different shapes and degrees of tilt, still seems too weak to explain the phenomenon described above. In any case, this field of research deserves attention: this phenomenon is one of the perceptual enigmas still awaiting a convincing explanation.

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