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Why anamorphoses look as they do: An experimental study *

Manfredo Massironi

University of Verona, Verona, Italy

Ugo Savardi

University of Padova, Padova, Italy

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The aim of this research was to understand why anamorphic images break up until they are unrecognizable when the observer's eye moves away from the regularization point. An experimental device was set up allowing the anamorphic deformation of images, consisting of a rotating screen on which figures were projected. The point from which subjects observed the screen was far from the projecting point. The projected figures lengthened equally when the screen rotated either clockwise or counterclockwise. On the other hand, the perceptual result was the opposite: in the former case, a rigid figure was seen rotating around its own vertical axis; in the latter case, the same figure was seen elongating or shortening in a non-rigid manner, without rotating. Since we were in a projective condition, the invariance of the cross-ratio was maintained. Therefore, we were in a situation of non-rigidity, in spite of the invariance of the cross-ratio. Three stimuli, white on a black background, were used in experiment 1. They were a segment, three aligned points, and four aligned points. Subjects had to rotate the screen at will and stop it at the point when they saw the transformation of movement from rigid rotation to non-rigid elongation. The results showed that: (i) in spite of being a projective invariant, the cross-ratio is not always a perceptual invariant too; (ii) the threshold screen position between the two motions was located at the position where the modifications of the solid angle subtended to the stimulus assumed a different trend from that of a sinusoid. Two stimuli were used in experiment 2: a continuous segment and one intersected by four vertical lines. The aim was the same as for experiment 1, but subjects had to repeat it from five different points of observation. The results showed that: (1) there was no significant difference depending on type of stimulus, indicating that the computability of the cross-ratio is not a necessary condition for the execution of the task; (2) the more the observer moved away from the projection axis, the more evident the distortion of the stimulus appeared, in accordance

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Requests for reprints should be sent to M. Massironi, Istituto di Psicologia, Università di Verona, vicolo cieco dietro S. Francesco, 37129 Verona, Italy.

with what happens when observing an anamorphosis; (3) when the metamorphosis from rigid motion to elastic motion was seen the visual angle subtended to the stimulus was constant for all distances from the projection axis, in accordance with the hypothesis of rigidity.

For sorrow's eyes, glazed with blinding tears,
Divides one thing entire to many objects,
Like perspectives, which rightly gaz'd upon
Show nothing but confusion; ey'd awry
Distinguish form;...

(W. Shakespeare, *Richard II* (II, ii, vv. 146 ss.))

One of the most problematic aspects of perception is related to the experience of stability and constancy of objects with respect to the variability of sensorial records.

The most significant approaches made by psychological literature may be divided into three groups: (i) theories which refer to the organizational processes of the cognitive system (Köhler 1929; Koffka 1935); (ii) theories which find possible explanations to the phenomenon in the empirical experiences of observers (Kilpatrick 1961) and in their inferential abilities (von Helmholtz 1911; Gregory 1966; Rock 1983); (iii) direct experience theories according to which the optical light spectrum affecting the eye is sufficiently structured to convey all necessary information regarding the structure and rigidity of objects (Gibson 1950, 1979; Johansson 1978; Cutting 1986).

One of the few conditions of experience in which the observed world seems to break up may be found in the observation of anamorphosis. Post-Renaissance perspective painters used anamorphosis on the one hand for the pleasure of wonder and the taste for deception and, on the other, to raise doubts and problems about the inadequacy of the senses and the unintelligibility of the world (Baltrusaitis 1955).

The most classical anamorphosis obtains an image by projecting its outline on a plane orthogonal to the projection axis on to another plane which is not parallel to the former. The more the second plane is tilted, the more deformed and widened does the image projected on to it become. Anamorphoses may also be obtained by projecting images on surfaces of different shapes (pyramidal, cylindrical, conical), and the Baroque painters used all these ploys (see Baltrusaitis 1955; Gardner

1975). In any case, the optical-geometric principle on which the perceptual effect is based is substantially the same.

The point from which the image is projected is the point of regularization of the figure. If the plane on which the anamorphosis is traced is observed from a parallel frontal vantage point, the figure will be less recognizable beacause the lines which define the objects are deformed and broken up. Inversely, if the anamorphosis plane is placed

Fig. 1. Vexierbild by E. Schon, app. 1535 (from Baltrusaitis 1955). When observed with one eye, keeping the sheet almost parallel to the optical axis, the drawing shows the portraits of four hystorical personages, three of which are Ferdinand I, Paul II and Francis I. Two of these, rectified, are shown in the lower images.

sagittally to the observer's optical axis and the figure is observed, even with only one eye, at the regularization point, the observer no longer has the impression of facing the rigid object. The various points of the figure no longer appear to be coherently organized but rather disjointed, as if they belong to elastic planes (fig. 1).

One may wonder why, when observing an anamorphosis, the slightest shift in relation to the regularization point produces a disjointed image, while for non-anamorphic images this does not happen. In the latter, the depicted objects and the whole scene do not appear deformed, even when observed from various tilted positions in relation to the normal frontal vantage point.

Kubovy (1986) defines as 'robustness of perspective' the fact that the properties of perspective representations do not induce any notable distortion effects in the observer when they are observed from points of view not coinciding with the projective centre geometrically used for their construction. Kubovy (1986: 52–64) discusses the theories regarding this problem and suggests a possible explanation.

In a systematic set of experiments Cutting (1987b) verified that, in spite of projective predictions, three-dimensional objects seldom appear non-rigid. The stimuli he used were cubes rotating around their vertical axis, displayed on a screen tilted at various angles. Results showed that, although objects were seen as rigid at small screen inclinations, they appeared as non-rigid at greater inclinations (45°).

The observation of anamorphic images by a moving observer produces the perception of elastic stretching of the image. According to us, this perceptual result appears to be closely related to the shape constancy of the observed object. Furthermore, the fact that anamorphic images are produced through operations of projections and sections indicates that they may be useful in the study of projective invariants in perception.

The well-known ecological theory of perception states that all visual information necessary for an animal to move and act successfully in its environment is conveyed by the optical flow, understood as the bundle of rigid angles subtended from the various points of environment to the observer's eyes.

As far as constancy of shape and therefore object rigidity is concerned, according to this theory, information is supposedly given by what remains constant in optical mediation, disregarding any present transformations depending on possible and/or different points of view

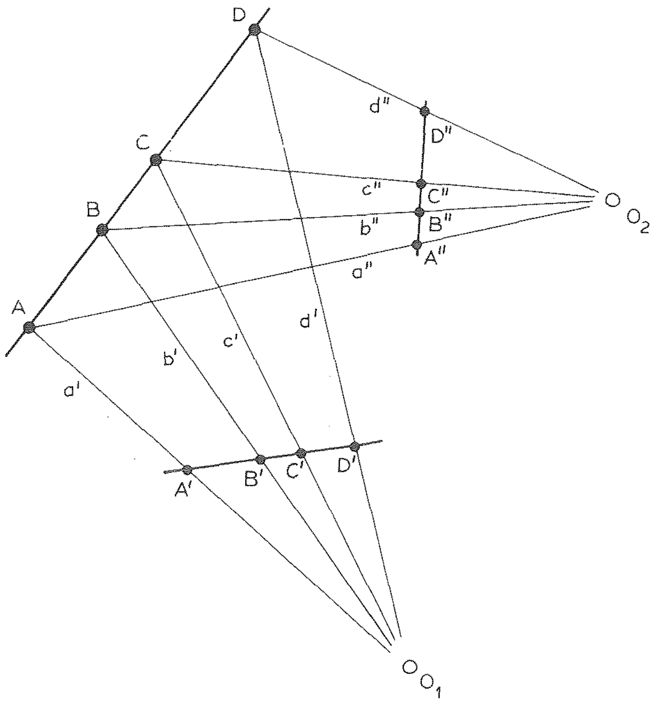


Fig. 2. The optical geometrical meaning of the cross-ratio. The cross-ratio between angular sizes (*abcd*) remains unchanged for all positions of *O* in relation to the configuration formed of the four points *A*, *B*, *C*, *D*. Equally, the cross-ratio between correspondent linear sizes (*ABCD*) forming on an arbitrary section plan also remains unchanged.

of the same object. Gibson (1979) defines as invariants all those geometrical and non-geometrical aspects which remain unchanged in the optical flow even when the circumstances of observation change. One basic projecting invariant is the cross-ratio (see fig. 2).

The cross-ratio (CR) has been adopted by the supporters of direct perception (Gibson 1979; Gibson et al. 1978; Johansson 1974; Johansonn et al. 1980; Shaw and Pittenger 1977; Michaels and Carello 1981; Hagen 1986; Gerbino 1983; Cutting 1986, 1987a, 1987b) as one of the most reliable sources of information as regards the rigidity of objects. According to this approach, when rigidity ruptures take place in the examined object, the particular relationships between points in the same object, known as cross-ratios, do not remain unchanged. Thus, information about changes in the examined structure may at once be deduced. In the light of these observations, as far as anamorphosis is concerned, a variation in the CR, according to the observer's point of view would be expected. Anamorphoses are projective constructions since, as we have seen, they are obtained by projecting the points of an image drawn on a plane, from a point external to that plane and cutting the bundle of rays thus obtained with a new plane, not parallel to the first. In such an operation, the CR between any set of four points remains unchanged. It is important to note that the CR between four collinear points, calculated from the regularization point (when the image appears to be well-organized and regular), has the same value as it does when the vantage point is far from the latter and the image is broken up and unrecognizable. If CR invariance were perceptually determinant information, the two images should turn out to be equivalent, which does not happen.

Our study had three aims:

(1) To perfect equipment based on the principle of anamorphic modifications, allowing experimental study of 'the boundary conditions of perceived non-rigidity over rigidity' (Cutting 1987a: 74).
(2) To understand why rigid configurations, like anamorphoses, may produce an elastic and non-rigid perceptual result in observers.
(3) To verify the actual perceptual usefulness of the CR in accordance with Cutting's statement (1986: p. 70), 'At best invariants work for perceivers only some of the time, and the determination of how well they work for psychologists is an empirical question'. Thus, our aim is to carry out empirical determination of the perceptive functions of the CR in certain controlled conditions.

For this purpose, a special device was set up allowing the fundamental successions of anamorphic transformations to be produced and empirically controlled.

Equipment

The equipment consists of a 60 × 50 cm rectangular surface mounted on a vertical pivot, connected to an electric motor, allowing clockwise and counterclockwise rotation of the plane.

The motor rotated at one revolution per minute. A protractor measuring the size of the rotation angle was fixed on the frame supporting the rotating plane. Two pairs of push-buttons were used to control plane rotation, one for clockwise and the other for counterclockwise rotation. One pair was for the experimenter and the other for the subject.

The configurations to be examined were projected on the above mentioned plane (hereafter called screen) by means of a normal slide projector.

The optical axis of the subject and the projector converged on the centre of the screen, forming an angle of 30°. The distance of the projector focus from the centre of the screen was 185 cm; the distance of the observer from the centre of the screen was 165 cm. (see fig. 3). When the screen was orthogonal to the projection axis, the protractor indicated 90° ('90° screen' indicates this condition of orthogonality).

If we consider the slide inserted in the projector as the starting image, orthogonal to the projection axis, the screen shows an image that becomes increasingly deformed the more the screen is rotated clockwise or counterclockwise, moving away from orthogonality with respect to the projection axis. Thus, each position of the screen that is not orthogonal to the projection axis shows an anamorphosis of the projection increasingly deformed with increasing screen inclination. This is a different condition from a classical anamorphosis, because an anamorphosis is the stable projective deformation of an image perceived by a perceiver moving past it. In our case, the observer remains stationary and observes anamorphic transformations that are constantly more accentuated from an angle of 30° (see fig. 3).

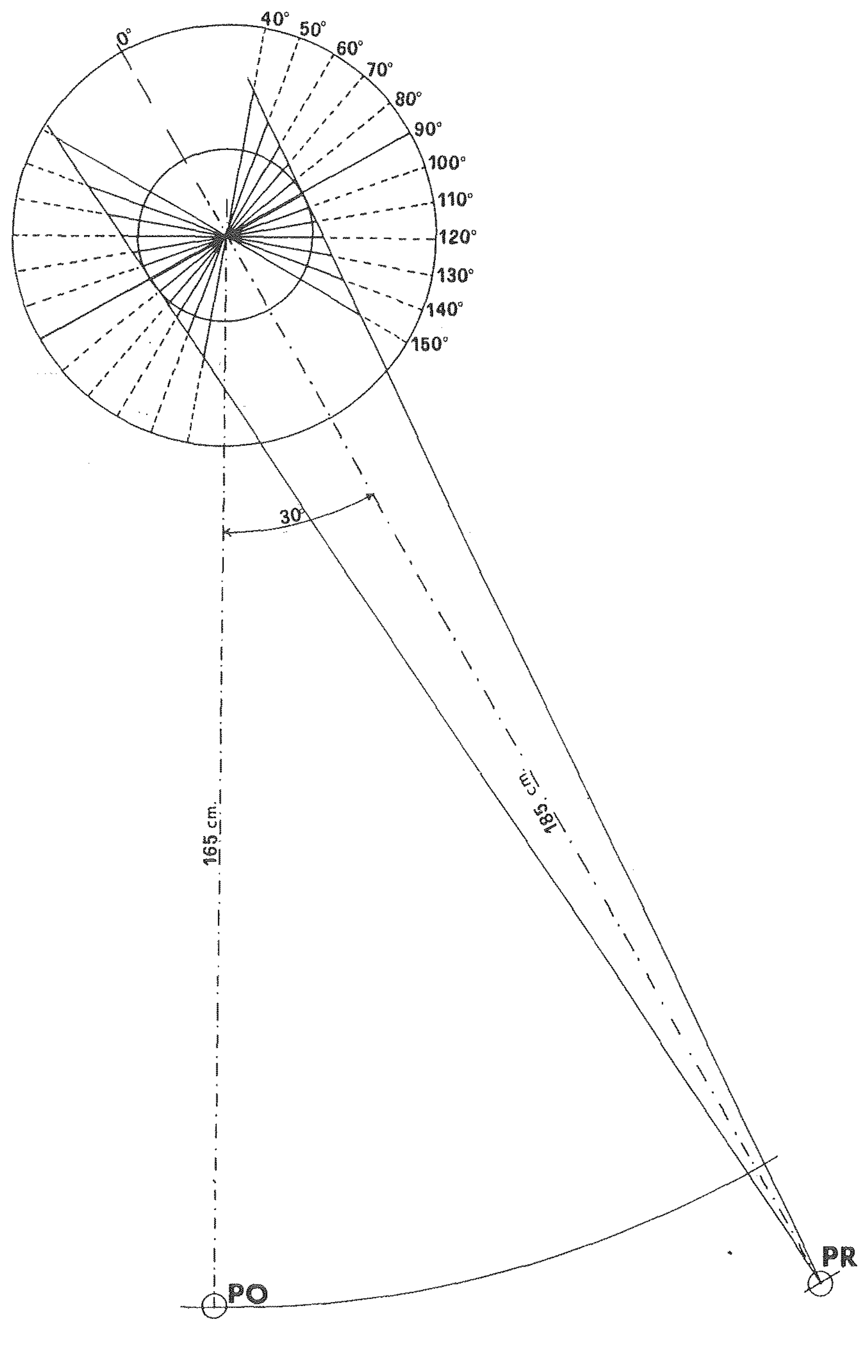
Preliminary phenomenic observations

Preliminary observations were carried out by five researchers of the Department of Psychology of the University of Padova, over various sessions. Several specially constructed stimuli, either geometrical figures or reproductions of classical anamorphoses, were projected on the screen. For each stimulus an accurate analysis of phenomenon, shape or movement changes, depending on clockwise or counterclockwise rotation of the screen, was performed.

As shown in fig. 3, according to the position of the screen a projected segment (St) undergoes elongation of 58% in the 40° and 140° positions. When subjects observe from point PO of fig. 3, the modification of the stimuli produced when rotating the screen suggests two main perceptual phenomena in contrast with each other: (1) When the screen was rotated back and forth from a sagittal position with respect to the observer (40° screen) to a point more or less orthogonal to the projection axis (90° screen), subjects stated that they saw a rigid object with constant dimensions which rotated back and forth. This evidence seems of vital theoretical importance, because it highlights a case in which non-rigid motion mimics rigid motion. (2) When the screen was rotated back and forth from approximately the 90° position to the 140° one (see fig. 3), subjects said they saw an non-rigid object, without rotation, which was elongated or shortened. These main phenomena were accompanied by other minor impressions, such as the approach of the stimulus towards the subject or temporary inversion of the direction of rotation.

Two experiments were set up with the aim of:

(1) Measuring in which position of the screen, during its continuous motion, the observer perceived the passage from rigid to non-rigid perception of the stimulus, in spite of the fact that the two physical deformations were identical.



0°
40°
50°
60°
70°
80°
90°
100°
110°
120°
130°
140°
150°
30°
165 cm.
185 cm.
PO
PR

(2) Establishing if this screen position was influenced by the observer's position.
(3) Supplying evidence to interpret this phenomenon.
(4) Supplying evidence to explain the perception of anamorphosis, even when considering that, with respect to classical anamorphoses, our situation was different.

Experiment 1

Method

The aim of experiment 1 was to verify in which position the screen was when the transformation from one movement to the other was seen. This seemed to be the first piece of information on which to base analysis of the conditions causing the phenomenon.

The equipment of fig. 3 was placed in a totally dark room. The projector screen was covered in matt black paper to reduce reflections. The stimuli were black slides in which the image was negative. A reduction screen was placed between the rotating screen and the observer so that only the stimulus could be seen.

Light from the projector was restricted so that subjects could only see the projected stimuli, never the surface of the screen. The stimuli were observed with both eyes. The rotating screen could move, 50° clockwise and 50° counterclockwise (see fig. 3), in relation to the condition of 90° (orthognal to the projection axis). Three stimuli slides were used, with the following features:

(1) A line which, when measured on the screen at 90°, was 272 mm long and 10 mm thick; at 140° and 40°, it became 430 mm long (see fig. 3);
(2) three aligned dots, equidistant from each other, with a diameter of 12 mm and a total distance of 222 mm at 90°. When the screen was positioned at 40° and 140°, the distance between the two outer points was 345 mm;
(3) four aligned dots, equidistant from each other, with a diameter of 12 mm and a total distance of 272 mm, at 90°. When the screen was positioned at 40° and 140°, the distance between the two outer points was 430 mm.

Subjects and procedure

Twenty subjects were chosen randomly among students and researchers of the Department of Psychology of the University of Padova. After a period of adaptation to darkness (10 min), subjects were made familiar with the use of the control push-buttons, and were then asked to rotate the screen as they liked in both directions and to describe what they saw. During these preliminary observations, subjects almost always

Fig. 3. Schematic representation of experimental situation plan. Large circle: screen rotation limits. Small circle: rotation limits of a rigid stimulus 272 mm long. Heavy line: experimental stimulus 272 mm long projected on screen in 90° position. Thin continuous lines: deformation undergone by experimental stimulus, projected on screen in 12 positions between 40° and 150°. PR: Projector's position. PO: Observer's position.

spontaneously noticed the existence of two different types of movement, as previously described. After this training period, lasting about 10 min, the real experiment started. The experimenter arranged the screen in one of the possible positions between 90° and 40° for 5 out of 10 tests, and between 90° and 140° for the other 5 tests; this variable is called 'direction'. The sequence of the positions was determined randomly and was different for each subject.

Subjects could not see the equipment while the experimenter adjusted the screen position.

The task consisted of moving the screen to and fro freely, stopping it when it seemed to change the type of perceived movement. The rotation speed of the screen was one revolution per minute, so that the task could be performed comfortably.

The method of subjective adjustment provided for 10 presentations of the line stimuli and 10 presentations of the point stimuli. The threshold value, with an approximation of 1 degree, was detected by a goniometer on the back of the screen.

Results and Discussion

The threshold value, the mean of the responses of all subjects, was 101°13′ ($SD = 8.42$) for the line, 100°54′ ($SD = 7.54$) for three aligned dots, and 102°56′ ($SD = 7.18$) for four aligned dots.

In order to verify the role of the independent variables, a two-way (direction and type of stimulus) within-subjects ANOVA was carried out. None of the variables or their interactions was statistically significant.

The only feature that is noticeable from the data is the considerable homogeneity of subjects' answers, which allowed constant operations by all subjects while carrying out the task to be assumed.

Projective geometry states that the CR remains unchanged within all projection and sectioning operations. These operations inevitably produce transformations, in both shape and size, of the projected and sectioned figures. From the perceptual point of view, the ecological approach states that, even when these transformations physically occur, if the CR remains unchanged, a perceptual result of constancy (and therefore a perceptual result of rigidity of the observed object) is produced. In our case this does not happen.

At first sight, it would seem that, while performing our task, subjects do not really base their responses on projective types of information such as the CR, but use a trigonometric type of information such as the visual angle subtended to the stimulus and its inclination with respect to the optical axis and its variations with time. While the CR between four points did not change during screen rotation, what did definitely change was the width of the visul angles subtended from the stimulus to the observer's eye. We found that although the projected stimulus underwent the same change in length when the screen rotated between 40° and 90° as when it rotated between 90° and 150°, the perceptual result was the opposite: rigidity in the former case and non-rigidity in the latter. A trigonometric explanation of this phenomenon can only be sustained if the changes in the visual angles subtended from the stimulus to the vantage point follow a different trend, according to whether the screen rotates clockwise or counterclokwise, starting from a position at right angles to the projection axis.

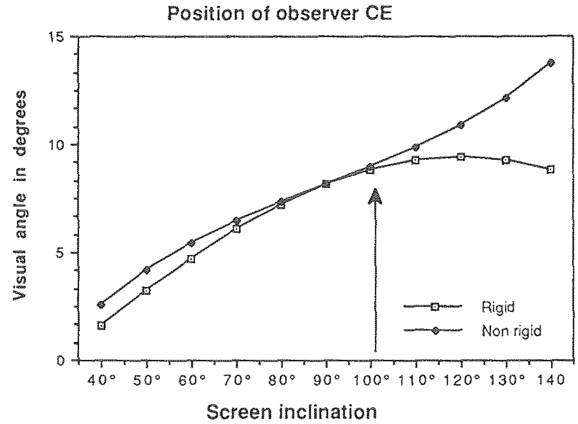


Fig. 4. Trends of visual angle modification relating to non-rigid experimental stimulus and a rigid theoretical stimulus of the same size, during rotation of screen between 40° and 140°.

With the aim of verifying this hypothesis, we calculated the visual angle for 11 positions of the stimulus, ranging between 40° and 140°, at intervals of 10°. This operation was only performed on the line: it was observed that the trend of the three stimuli was practically identical, and we therefore chose the line due to its greater simplicity of handling and because its threshold value was intermediate among the three conditions observed. Measurements were taken both for the line used in the experiment and for a theoretical line, of the same size but considered as undeformed throughout the rotation. The two curves of fig. 4 were obtained from these data. The curves have the same trend throughout rotation between 40° and 100°. When rotation exceeds 100°, the two curves start to diverge, clearly taking on opposite trends. In the adjustment tests, subjects showed that they could locate the point of divergence of these two curves with considerable accuracy.

Fig. 4 highlights a series of facts:

(1) The lower curve in fig. 4 shows the section of sinusoid that corresponds to a rigid bar 272 mm long fixed to the screen and rotating between 40° and 140°, observed from position PO of fig. 3. The visual angle subtended to our experimental stimulus is represented by the upper curve, the trend of which is very similar to a sinusoid when the screen rotates between 40° and 100°. Although the stimulus is elongated by 58%, the visual angle subtended to the eye of the observer progressively narrows, following a trend similar to that of a sinusoid for a rigid object rotating through a similar arc.

(2) The position of the screen at 100° corresponds to the mean position when subjects saw the passage from rigid to non-rigid motion. This screen position corresponds exactly to the position in which the upper curve starts to diverge from the lower curve.

(3) When the screen rotates between 90° and 140°, the projected stimulus is once again elongated by 58%, but in this case the visual angle, starting from 100°, progressively widens and constantly diverges from the trend of that subtended from the

rigid stimulus. The above data show a closer correspondence between perceptual result and change in visual angle than between perceptual result and CR. Perhaps, then, the former is more useful (and possibly more exploited) than the latter?

For a rotation arc between 40° and 100°, these changes seem to be congruent with those of a rigid rotating bar: this may induce the assumption of rigidity (Ullman 1984a, 1984b; Palmer 1983), so that subjects expect that the visual angle will undergo a specific change into a sinusoidal trend. As soon as the angle starts to diverge from the expected one, its rigidity is lost.

According to this point of view, the process underlying the phenomenon analysed provides for an initial assumption of stimulus rigidity, which may subsequently activate a scheme (Neisser 1967; Norman 1979; Garner 1981), in turn providing for and controlling the trend of the transformations of a rigid rod around its own axis. This scheme is maintained as long as it fits the data coming from the stimuli and is obviously abandoned when the fit disappears. This process occurs within the conditions in which the invariance of the CR is maintained.

On the other hand, Cutting (1987a) emphasizes that the assumption of rigidity, though at a hidden level, is already potentially present in Gibson's (1979) invariance and in Johansson's (1978) decoding principles.

It is not easy to explain the actual causes and information used for the production of the two perceptual states. The evidence which has emerged may be summarized as follows: (1) non-usage of calculation of the CR while executing the task; (2) assumption of rigidity deduced by the changing of the visual angle in time; (3) screen inclination at 100° as the position in which the transformation of movement from rigid to non-rigid occurs.

The aim of our second experiment was to further verify each of the above three points.

It should be specified that the threshold value (point between rigid rotation and plastic modification of the stimulus) has no absolute value and depends on the observer's position (PO in fig. 3) in relation to that of the projector (PR in fig. 3). If PO coincides with PR, we have the situation assumed by von Helmholtz (1911) in his theoretical experiment, where nothing changes in the observer's experience because all deformations occur along the lines of the same visual rays. This situation is also defined by the formula proposed by Ittelson and Kilpatrick (1961) to designate equivalent configurations.

Experiment 2

Let us summarize the hypothesis and aims of experiment 2.

(1) In experiment 1, probably none of the stimuli completely represented the conditions for CR calculation in the classical manner, that is, the presence of at least four identifiable points on a single-dimension stimulus. We assume that there is no significant difference between two linear, single-dimension stimuli of the same length, only one of which shows four perceptually identifiable points. Among others, von Staudt (1847) and Russell (1897) maintained that projective geometry is strictly

qualitative. In this case, CR invariance may be a quantitative aspect which accompanies projection and section operations and which can therefore only be revealed and calculated if we have four collinear points. But this does not mean that, when the CR cannot be calculated because the four points are not defined, projective properties do not apply.

We therefore believe that projective relations, also from a perceptual point of view, remain unchanged both when the four points allowing calculation of the CR are perceptually identifiable and when they are not. Our hypothesis is that there should be no difference between two situations of this type when they are studied with our equipment.

(2) The screen position at which the separation between the two types of movement (rigid and non-rigid) is perceived, may be said to depend (distance from the screen being equal) on the shift in the vantage point with respect to the projector. That is, the greater this shift, the greater the screen inclination at which non-rigid deformation is seen. Indeed, observing the stimulus from point PR in fig. 3 means observing it from the point of perceptual indetermination described by von Helmholtz (1911) and thus we do not perceive any change during screen rotation.

Fig. 6a was constructed with the same criterion used for fig. 4. The lower curve represents the modification of the visual angle subtended to the rigid object observed from the projection point (PR in fig. 3), whereas the upper line parallel to the X axis represents the visual angle subtended to the experimental stimulus. Comparing the graphs of fig. 6, and bearing in mind that fig. 6c is identical to fig. 4, we see that displacement of the observation point from PR towards the left causes the shape of the upper curve to change.

If the hypothesis of using trigonometric information associated with the assumption of rigidity (see discussion of experiment 1) is true, this should imply that, considering equidistance from the screen, for any observation point (excluding the regularization point) the visual angle subtended to the stimulus in which transformation from rigid to non-rigid movement occurs, should remain constant.

(3) In our opinion, the observer's greater or lesser distance from the screen should not influence the screen position at which the change in movement of the stimulus from rigid to non-rigid is perceived. This is because the angular shift of the observer's optical axis from the projection axis remains constant.

Method

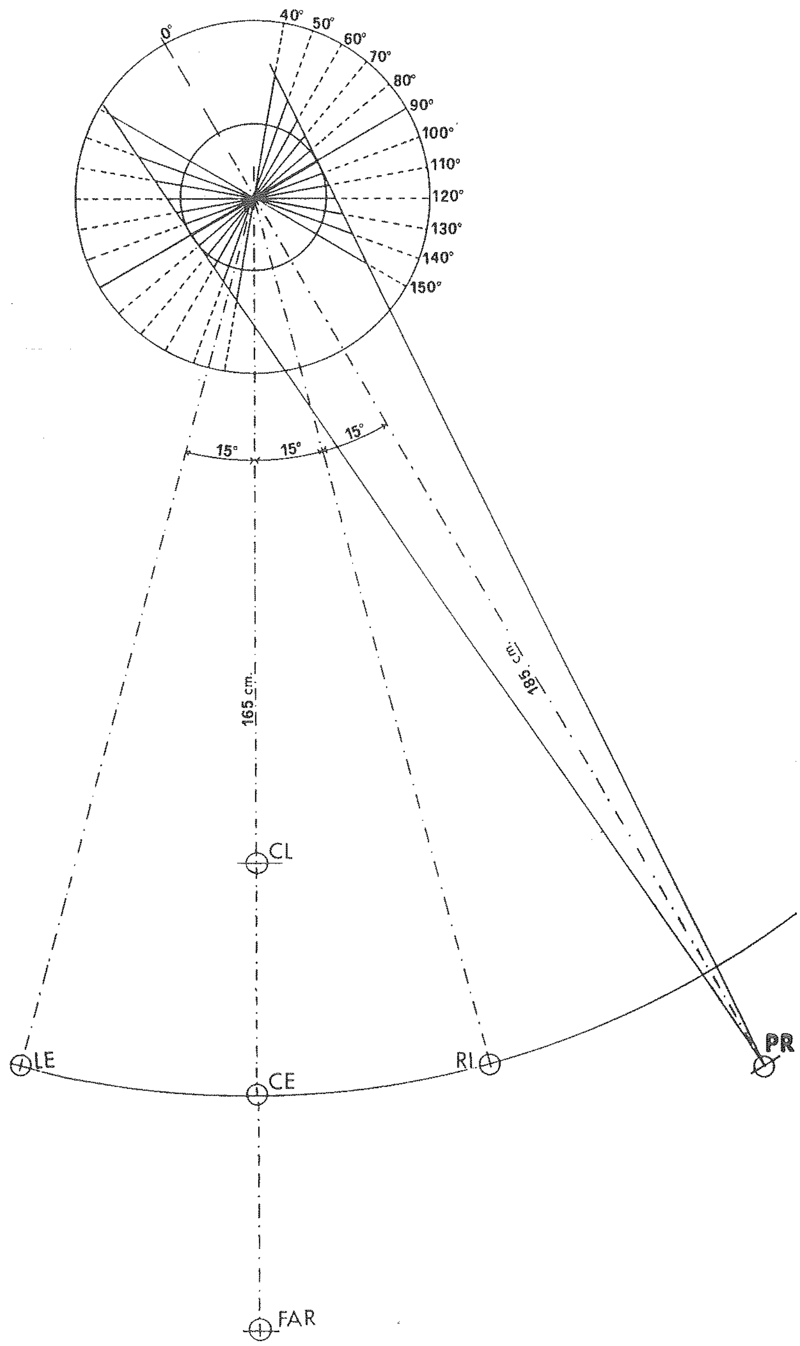
Stimuli

Two stimuli were used: (1) a horizontal straight segment of the same length as that used in experiment 1 (272 mm), but much thinner (4 mm) so as to make it as single-dimensional as possible; (2) another segment identical to the previous one, but intersected by' four vertical segments (15 mm), placed one on each end and the other two within the segment, equidistant from each other.

The first stimulus is called 'continuous line' and the second 'intersected line'.

Subjects

Subjects were 10 psychology students aged between 20 and 25, with no prior knowledge of the aim of the experiment.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15°
165 cm.
185 cm.
CL
LE
CE
RI
PR
FAR

Procedure

The equipment and observation conditions were the same as those in experiment 1. However, each subject had to repeat the test from 5 different positions, shown in fig. 5. CE (central) indicates the point of observation used in experiment 1, calculated at 165 cm from the centre of the screen and in such a way that the optical axis of the observer formed an angle of 30° with the projection axis. Another two points were set: RI (right) in which the optical axis of the observer formed an angle of 15° with the projection angle, and LE (left) in which the angle was 45° (see fig. 5).

The other two points, CL (close) and FAR, were set along the optical axis of the observer placed at point CE, one above and one below CE and at a distance of 43 cm from that point. Fig. 5 (heavy line) also shows the stimulus in the 90° position. The elongation of this stimulus is also visible for the various screen positions between 40° and 150°, at intervals of 10° (continuous thin lines).

Subjects were asked to adjust the screen for both stimuli and for all 5 observation points. The sequence was randomized and different for each subject, as regards both position and stimuli. Task arrangement and execution were the same as for experiment 1. It should be remembered that it was the position of the screen (rotated under subjects' control) in which the movement of a rigid rod, rotating around its own axis, appeared to be transformed into the non-rigid movement of an object which became elongated or shortened. Each subject had to perform 10 adjustments for each stimulus and position, for a total of 100 responses. The execution of the whole task lasted about 120 minutes per subject and was carried out in two sessions of about an hour each.

Results and Discussion

Each response by subjects stopped the screen in a certain position. The measurement of the angle indicated by the protractor, with point zero on the continuation of the projection axis (see fig. 3), was the value of each response (used for statistical analysis).

Effect of the two stimuli

The data obtained in this manner allowed calculation of a two-way ANOVA for repeated data in which two observation positions (5 levels) and the type of stimulus (2 levels) were compared. The main effect regarding the observation position turned out to be significant ($F(4,90) = 183.55$ $p < 0.005$). Neither the difference between the two stimuli nor the distance interaction for the stimuli was significant. This first result indicates how the position of the screen in which the passage from rigid to non-rigid motion is observed, changes according to the position of the observer. However, this result requires more detailed analysis (see below).

Fig. 5. Schematic representation of experimental set-up. Large circle: screen rotation limits. Small circle: rotation limits of a rigid stimulus 272 mm long. Heavy line: experimental stimulus 272 mm long projected on screen in 90° position. Thin continuous lines: deformation undergone by experimental stymulus, projected on screen in 12 positions between 40° and 150°. Points LE, CE, RI, CL, FAR, show the five positions from which subjects performed task in experiment 2.

For the time being, let us stress that no statistically significant difference emerged from our data, either when operating with one stimulus allowing the CR to be computed or with one which did not. Fig. 6 shows the trend of the two stimuli calculated on the mean responses obtained for the various observation points. The ordinate indicates screen inclination.

Observer's position

The effect of the observer's position emerging from the ANOVA does not appear to be important, because it depends on screen inclination referred to the projection axis and not to the observer's optical axis. Only the inclination angle of the screen referred

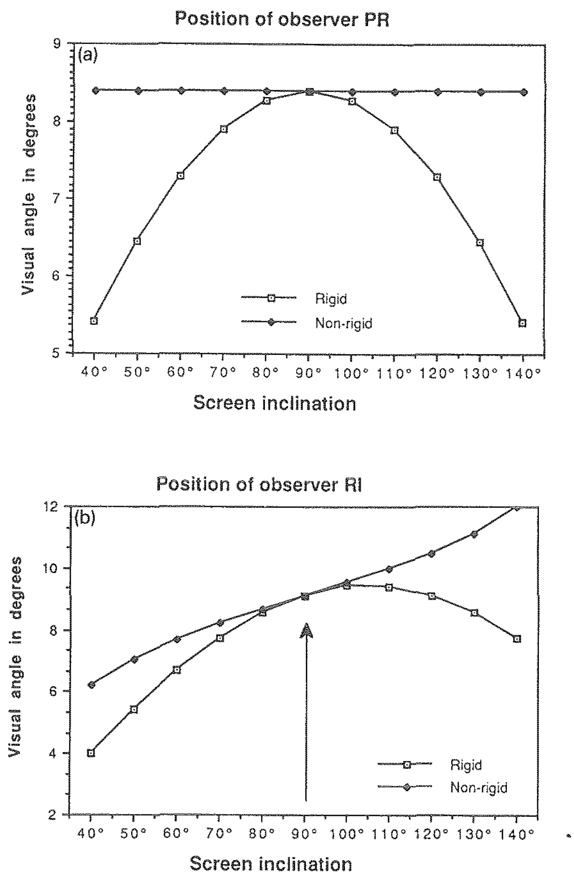
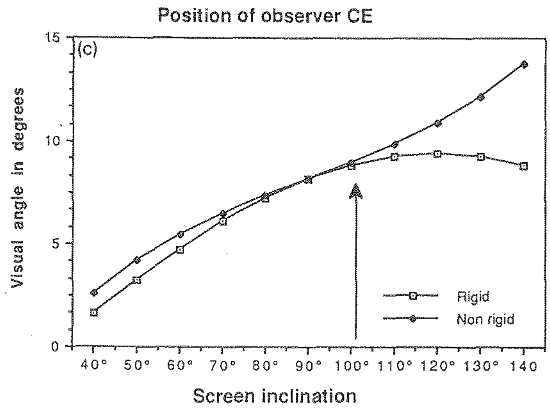


Fig. 6. The two curves of each diagram show trend of visual angles subtended to non-rigid experimental stimulus and a rigid theoretical stimulus of the same size for observation points PR (position of projector), RI (right), CE (central), LE (left). Arrows: inclination of screen when passage from rigid to non-rigid motion was observed.



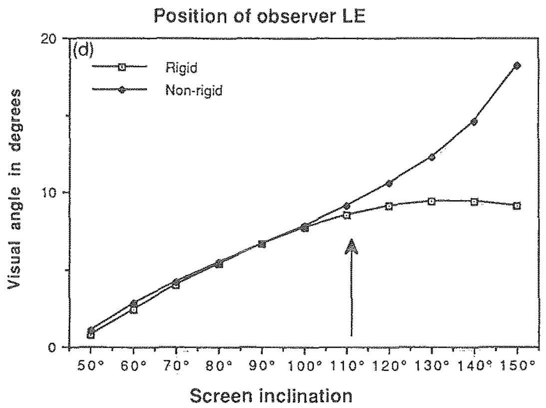


Fig. 6 (continued).

to the observer's optical axis (fig. 5, dotted lines) can indicate if, from the various points of view (RI, CE, LE), stimulus deformations are seen to the same extent. To verify this assumption, we equalized the data supplied by subjects. Keeping the projection axis as a reference point, LE was located 15° left of CE, whereas RI was located 15° right of CE (fig. 5). To compare the equalized data we subtracted 15° from each subject's response for LE and added 15° for each response for RI. Responses for CE were left unchanged, allowing us to verify the effect of the observation point in an absolute sense.

A two-way ANOVA (distance: 3 levels; stimulus: 2 levels) was computed. The effect of distance turned out to be significant ($F(2,54) = 49{,}69$ $p < 0.05$), whereas differences regarding type of stimulus and distance interactions were not significant.

The mean values for the new data show that, from LE, the difference between the two types of movements is observed at a screen inclination of less than 5° with respect

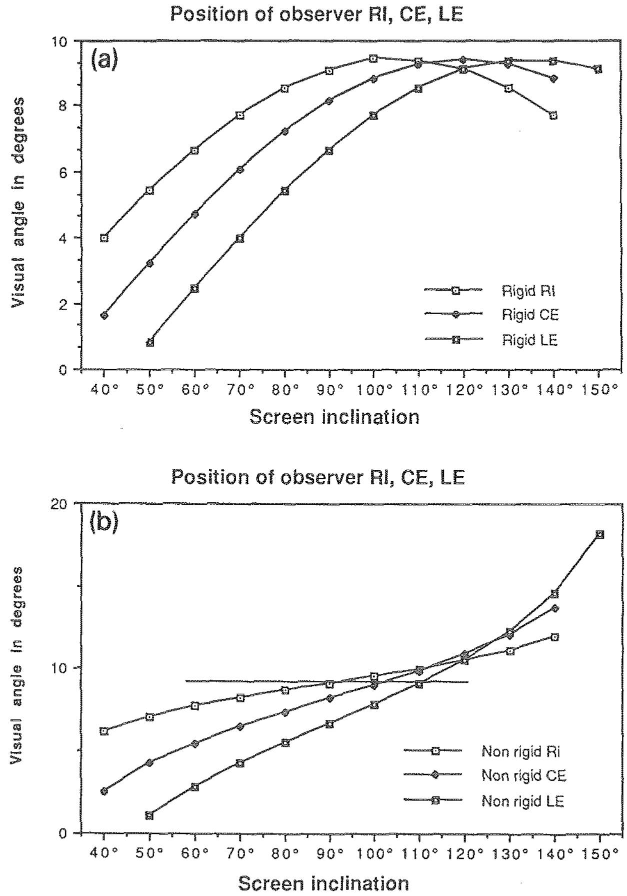


Fig. 7. (a) Curves of visual angle modification relative to a rigid stimulus 272 mm long observed from positions CE and LE. (b) Modifications relative to experimental stimulus observed from positions RI, CE and LE.

to CE and 10° with respect to RI. This means that non-rigid deformation of the stimulus is perceived early, during screen rotation, the more the vantage point is shifted from the projection axis.

An important effect explaining anamorphosis may be seen in fig. 7, which shows the three curves reflecting the trend of the visual angles subtended to the rigid stimulus observed from RI, CE, LE (fig. 7a), and the curves of the visual angle subtended to the projected stimulus for the same observation points (fig. 7b). The curves in fig. 7b are different in both shape and inclination, whereas those in fig. 7a are all the same, although they are 15° from each other.

The different inclination of the curves in fig. 7b shows that the visual angle subtended to the ends of the projected segment, increases the more the vantage point is removed from the projection axis ('non-rigid LE' in fig. 7b) and is greater when screen inclination is greater (trend of curves in right-hand part of fig. 7b).

This is why the painters of past centuries who developed anamorphosis chose very marked inclinations of the plane on which their images were projected and drawn, with the aim of puzzling their spectators.

Fig. 6 shows the increasing amplitudes of the single visual angle subtended to the experimental stimulus (S-shaped curves) and to a rigid segment of the same length (sinusoidal section). For RI and CE we considered the movement of the screen between 40° and 140°, whereas for LE we considered screen rotation between 50° and 150°, because the screen was no longer visible at 40° from that position (see fig. 5).

Each of the graphs in fig. 6 shows two curves, one relating to the trend of the visual angle subtended to the experimental stimulus (non-rigid), the other relating to the trend of a rigid stimulus of the same size, for the 11 screen positions shown on the abscissa. When the observation point moves from PR to LE, the first curve assumes a shape continuously more similar to the second on the left-hand side of the graphs, whereas the two curves constantly diverge on the right-hand side of the graphs.

The separation threshold of the two movements (rigid and non-rigid) is 90° when the observer is at RI, 100° when s/he is at CE, and 110° when s/he is at LE (see arrows in fig. 6). This shift causes the distance between the vertex of the sinusoid and the threshold point to remain practically constant. This result appears to be particularly important, because it indicates that the three screen positions at which the subjects see the metamorphosis occur, from rigid motion to non-rigid, subtend to a constant visual angle, the amplitude of which is close to that subtended to the rigid stimulus when it is in a frontal parallel position. The amplitude of the visual angle subtended to the experimental stimulus is 9°0.5′ for RI, 9° for CE and 9°0.7′ for LE (see three points connected by horizontal segment in fig. 7b).

These results suggest that, in our task, the perceptual system takes into account especially visual angle modifications in time. As regards the assumption of rigidity at all moments the modifications of the observed object are estimated, and their compatibility with that of a rigid object is verified. The modifications in time of the subtended visual angle to a rigid segment of 272 mm observed from a distance of 165 cm forecasts a maximum amplitude of 9°27′; as soon as this value is reached, the assumption of rigidity of a rotating object forecasts a single result: reduction of the visual angle. If this does not happen and it is increased instead, rigidity is abandoned and another type of movement – non-rigid – is seen.

Observing fig. 6b we may ask why subjects see the (non-rigid) experimental stimulus as rigid, rotating around its axis at a screen inclination between 40° and 90°, although the curve which describes it evidently moves away from the curve describing a truly rigid object of the same size. For the time being, we interpret this fact by pointing out that, between 40° and 90°, the curve in question approximates to a section of a sinusoid and thus that subjects probably see a segment that is slightly longer and rotates slightly more slowly, although we do not have data to confirm this hypothesis.

The effect of distance from screen

A two-way ANOVA was calculated for the repeated data, comparing the observer's distance from the screen (3 levels: CL, CE, FAR) and the type of stimulus (2 levels:

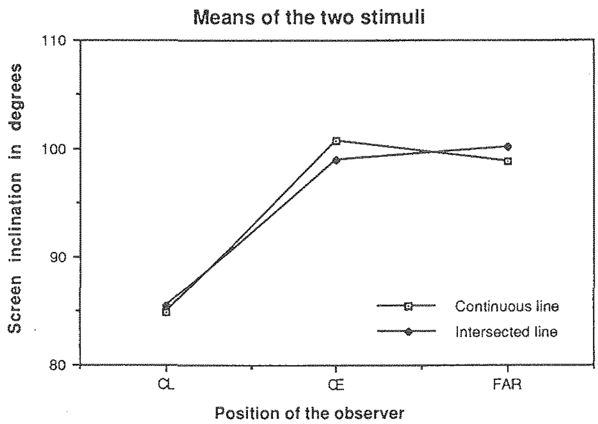


Fig. 8. Inclinations of screen at which transformation from rigid motion to non-rigid motion was seen, for three distances from screen indicated in abscissa (CL, CE, FAR).

continuous line, intersected line). The main effect for distance was significant ($F(2,54) = 102.57$ $p < 0.025$), whereas the effect of difference of stimuli and distance interaction between stimuli was not. Fig. 8 shows at which screen inclinations the metamorphosis of motion for the two stimuli was seen at the three distances.

While no difference emerged between the central (CE) and far (FAR) distances, in the close position (CL) the change in type of motion was seen at a lesser screen inclination. In our opinion, the trends of the curves confirm our previous observations regarding the observer's position: we should have expected that the position at which the screen was stopped during execution of the task (fig. 9: arrows) would be the same for all three distances. In fact for all three, the curves of the three graphs show similar trends. Moreover, at 100° screen inclination, the non-rigid stimulus curve begins to diverge from the rigid stimulus curve, for all three distances.

It may be established from this that the curves relating to the non-rigid and rigid stimuli start diverging at a screen inclination of 100° for all three distances. However, while 100° is confirmed by subjects' responses for CE and FAR, it is reduced to about 85% for CL. We do not have a definite explanation for this result, unless it is an optical consequence due to the nearness of the stimulus.

As partial support for this hypothesis, we verified that the amplitude of the stimulus, when the metamorphosis between the two kinds of movement from all the other positions (RI, CE, LE, FAR) is seen, implies foveal vision: in fact, it always subtended a visual angle of less than 10°. Instead, in CL, a parafoveal half-tone contact screen was intersected, which probably expanded the enlargement effect of the stimulus when the screen rotated beyond 90°, causing a backward correction of screen inclination during the various adjustments which subjects could make.

Partial conclusions

The main results of the work performed up to this point may be summarized as follows:

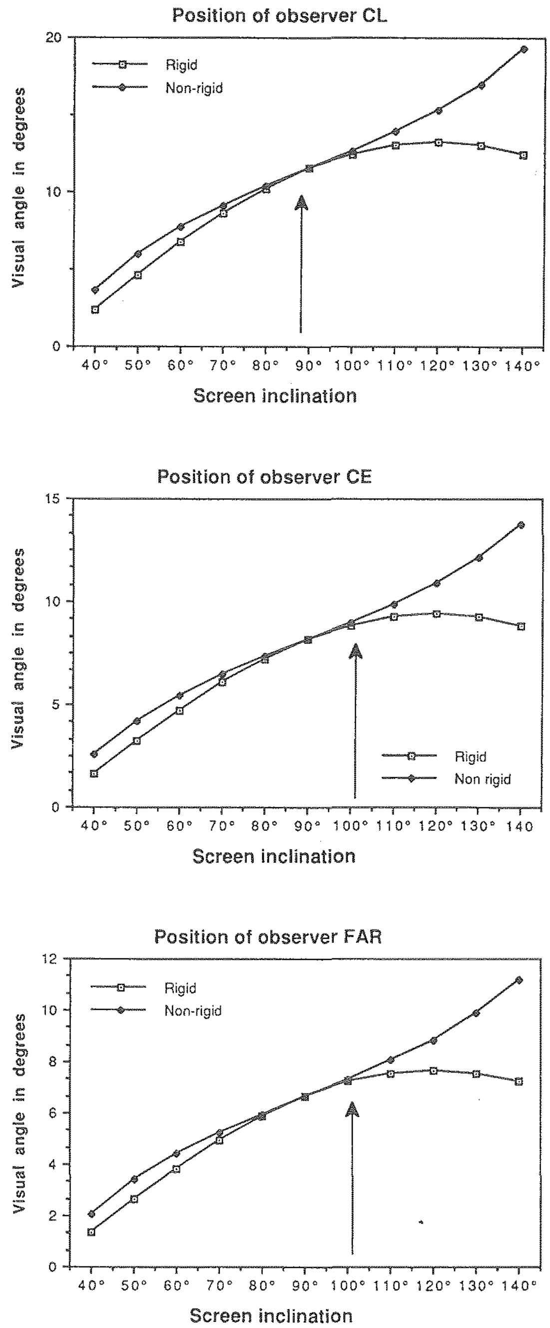


Fig. 9. The two curves of each diagram show trend of visual angles subtended to non-rigid experimental stimulus and to a rigid theoretical stimulus of the same size for observation point CL (close), CE (central) and FAR. Arrows: inclination of the screen when passage from rigid to non-rigid motion was seen.

(1) Of the stimuli used in the two experiments, some allowed calculation of CR (four aligned dots, intersected line) and others did not (3 dots, continuous line). We have seen that no significant difference emerged among these stimuli during task execution. This may indicate that CR invariance is not determinant for perceptual discrimination between the rigidity and non-rigidity of an observed object.

(2) The perceived width of stimulus deformation is a function of the shift in the vantage point with respect to the projection axis. When the screen is rotated between 90° and 140°, this shift produces a greater change in the visual angle subtended to the ends of the stimulus (compare curves of 'non-rigid' stimulus in figs. 6b, 6c and 6d, bearing in mind the different scale shown on the *Y* axis).

(3) At an equal distance from the screen, the visual angle subtended to the stimulus when the metamorphosis from rigid to non-rigid motion is seen, is constant for all shifts from the projection axis. This result is in accordance with the assumption of rigidity, which allows us to evaluate and control, at all times, the compatibility of modifications of the subtended visual angle to the experimental stimulus with the modifications the rigid object undergoes when it rotates.

This is an important result if we remember that the inclination of the relevant curves to the non-rigid stimuli change inclination and shape according to the distance of the vantage point from the projection axis.

Final conclusions

On various occasions we have tried to use 'our experimental equipment in ecologically richer observation conditions than those applied during these experiments. In these conditions, the environment in which the test was performed was simply in shadow and thus the edges of the screen, like the other objects in the room, were clearly visible, and the two types of motion, rigid and non-rigid, were also clearly perceived. Rigid motion was seen as a segment drawn on the screen and rotating in unison with it, whereas non-rigid motion was seen as a segment which slid over the surface of the screen, elongating as the screen rotated. This was enhanced by the fact that observers could see the right and left edges of the screen, and had further information on the perception of non-rigid movement: apart from seeing the stimulus elongating, they were also able to see the space between the ends of the stimulus and the left and right edges of the screen become progressively smaller. These facts indicate that the phenomenon is definitely strong and perceptually coercive; it consists of one of those effects that, as Todd (1985) says, are imposed on the observer even when they are 'contaminated by large amounts of visual noise'. The experimental constraints were in fact chosen so as not to highlight the phenomenon, but to guarantee greater control over the variables examined.

Our analysis of the results emerging from these experiments, although it may appear to be an explanation, is in fact only a description of the phenomenon. Our identification of how and which visual components intervene in the perceptual events produced by our equipment and our attempt at explaining them by means of the assumption of rigidity do not yet provide a satisfactory theoretical basis.

However, in our opinion, one problem does seem to be well clarified: why anamorphic images deform and break up more than non-anamorphic images when the observer moves away from the regularization point. This clarification is based on a geometric point of view, which demonstrates how, in anamorphosis, moving away from the regularization point causes a different modification of visual information with respect to what happens with a non-anamorphic image. In order to understand how this result emerges from the data of our experiment 2, we must examine fig. 7 again from another viewpoint, stressing the differences between observing an image which is anamorphic and one which is not.

(a) Observation of a non-anamorphic image. A non-anamorphic image is one in which any figure, e.g., segment AB of fig. 10a lying on plane α, is projected on plane α', parallel to α. (This somewhat laborious description of a non-anamorphic image is used in analogy with our description regarding anamorphoses; see below.) If we look at segment $A'B'$ from points a, b, c, d, etc., the changes in the visual angle subtended from our eyes to the ends of $A'B'$ will follow a sinusoidal trend, like that shown in the curves of fig. 7a.

(b) Observation of an anamorphic image. An anamorphic image is obtained when any figure, e.g., segment CD lying on plane β is projected on plane β', not parallel to β (fig. 10b). If we look at segment $C'D'$ from points a, b, c, d, etc., the changes in the visul angle subtended from our eyes to the ends of $C'D'$ will follow a trend like that shown by the curves in fig. 7b. The more β' is inclined with respect to β, the more the shape and inclination of these curves move away from a sinusoid. If we compare figs. 10a and 10b we see that, from the optical viewpoint, there is no difference between AB and $A'B'$, whereas there is a large difference between CD and $C'D'$.

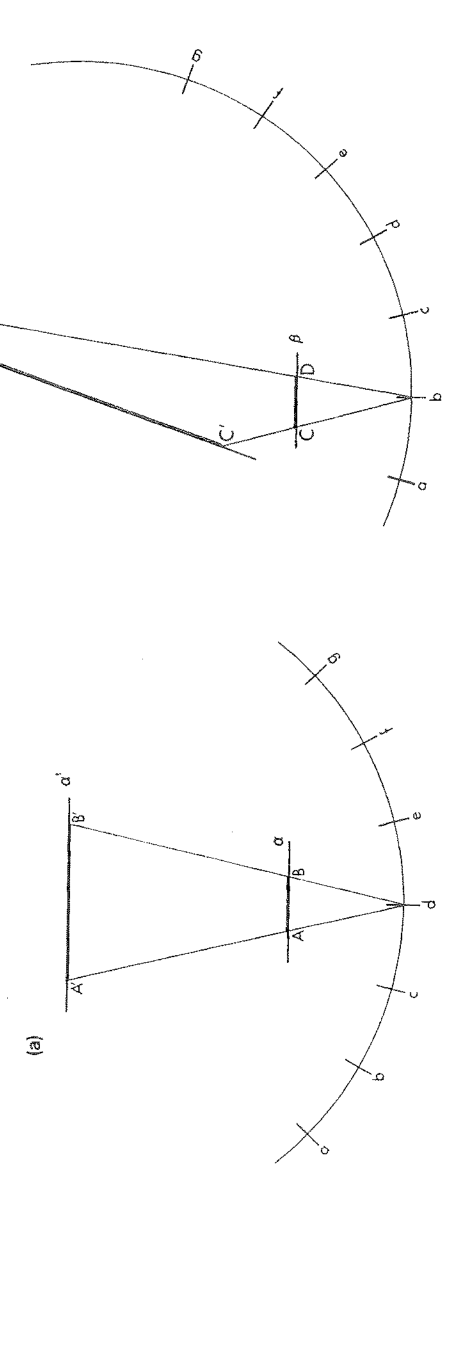


Fig. 10. Differences in observation of (a) non-anamorphic and (b) anamorphic images. See text for explanation.

Obviously, the positions from which segment $C'D'$ may be observed all tend to be to one side of the point of projection. This is why the Baroque painters, in order to increase the deformational effect, constructed anamorphoses whose regularization point was almost sagittal with respect to the projection axis (see fig. 1). In these conditions, there is only one direction in wich the observer may move, and that – in the case of fig. 10b – is to the right of the projection point. However, this very move is what produces a trend in the change of the visual angle which, in turn, moves from the sinusoidal form. This fact is clearly shown by the upper half of the curves in fig. 7b. From the optical viewpoint, the reasons for the differences between a normal and an anamorphic image thus seem to lie in the fact that, in the former, the change in visual angle subtended to two rigid points of the image has a sinusoidal trend, while in the latter it does not.

On the basis of this explanation, the visual information on which anamorphic effects are based is transmitted by the optic flow without the intervention of inferential processes, as previously hypothesized (Pirenne 1970).

In our opinion, the most important and probably most significant aspect of our work consists of the phenomenon which our equipment highlighted, i.e.:

(1) A perceptual event in which a rigid transformation mimics a non-rigid one. This has been thought, if not impossible, at least improbable by researchers referring to the ecological approach.

(2) A physical flow of continuous modifications which split into two different events in the perceptual experience of the observer (rigid rotation and non-rigid deformation). Thus, a boundary condition between the perception of rigid and non-rigid modifications is created. As this has not been deeply studied, it may supply new information on event perception.

(3) A condition producing the perception of rigid and non-rigid modifications, again within the invariance interval of the CR. The results obtained agree with those of Niall (1987), 'as it is, perhaps the cross ratio is never a psychologically effective invariant'. For this reason, our experimental equipment seems to be appropriate for redefining the projective invariants effectively used in perception.

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