

THE VISUAL STRUCTURE OF EVEN AND ODD. PERCEPTUAL ORIGIN OF THEIR OPPOSITION.

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Sommario

La ricerca ha studiato il ruolo dei fattori percettivi nella definizione piagorica di pari e dispari come numeri opposti. Lo scopo era di indagare se questa opposizione si basa su precise proprietà visive dei numeri-figura usati dai pitagorici (gli gnomoni).

Le 10 coppie di opposti indicate dai Pitagorici e riportate da Aristotele (*Metafisica* A5, 985b, 22) sono state usate come scale di un differenziale semantico. 66 studenti universitari hanno partecipato alla ricerca. Tre variabili sono state studiate entro i soggetti: le parole “pari” e “dispari”; matrici di punti rettangolare e quadrata, rappresentazione dei numeri pitagorici quadrati e rettangoli; margini rettangolare e quadrato, rappresentazioni dei due segmenti ortogonali componenti lo gnomone. I risultati dimostrano che la descrizione semantica delle parole “pari” e “dispari”, entro le 10 coppie, risulta effettivamente opposta, ma invertita rispetto a quella prevista dai Pitagorici (pari é polarizzato verso i poli positivi, dispari verso quelli negativi), confermando l’influenza di fattori culturali. Quando la descrizione è basata sulle strutture visive, l’opposizione risulta visibile solo nei margini gnomonici e non con le matrici di punti. Si sostiene che questo risultato supporta sperimentalmente l’ipotesi (Zellini, 1996) che l’origine del numero sia nell’idea greca di rapporto inteso come *diastema* (rapporto tra lunghezze) più che nell’idea di rapporto come *logos* (rapporto tra quantità discrete).

Abstract

The research studied the role of perceptual factors in Pythagorean definition of *even* and *odd* as opposite numbers. The purpose was to analyze whether this opposition is based upon precise visual features of the Pythagorean figural numbers (gnomons).

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The ten pairs of opposites used by the Pythagoreans and reported by Aristotle (*Metaphysics* A5, 985b, 22) were used as scales of a semantic differential. Participants were 66 undergraduate students. Three conditions were studied within subjects: words “odd” and “even”; rectangular and squared dot lattices, representations of the square and oblong Pythagorean numbers; rectangular and squared margins, representations of the two right angled segments composing the gnomon. The results demonstrate that the semantic description of even and odd, as words, turn out to be opposite in the ten scales, but in an inverted direction with respect to the one predicted by Pythagoreans (*even* is polarized in the positive sides of the scales, while *odd* is in the negative), confirming the influence of cultural factors. When the description is based on the visual structures, the opposition turned out to be visible only in the gnomonic margins and not at the dot lattices. It is argued that this result experimentally supports the hypothesis (Zellini, 1996) that the origin of number is in the Greek idea of ratio as *diastema* (ratio between length), better than in the idea of ratio as *logos* (ratio between discrete quantity).

1. Introduction

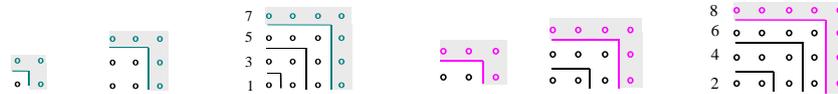
Within the framework of the study of perception in the Gestalt psychology tradition, a notable relevance was accorded to the theoretical and experimental analysis of relations such as unity (Kohler 1929; Wertheimer 1923; Ternus 1926), identity (Wertheimer 1912, Bozzi 1969), similarity (Goldmeier 1937, Tversky 1977), and causality (Michotte, 1954). The general claim that we suggest and that is guiding our research directions (Savardi and Bianchi, 1997, 2000) is that the study of the forementioned perceptual relations must take into account the perception of opposition, as the other face of the coin. The basic assumption is that opposition is perceivable between two objects or properties, in the same sense in which phenomenologists assumed the perceptibility of unity, causality, and identity, even though opposition does not appear in the classical research in this field.

The relevance of opposing properties in organizing our perceptual experiences was clearly described in pre-socratic philosophy, where opposites were presented as principles of the existent world and where the opposition between even and odd numbers appeared for the first time. It was introduced by the Pythagoreans in a list of 10 opposites (Aristotle, *Metaphysics*, A 5 985b 22): 1) limited and unlimited, 2) odd and even, 3) one and plurality,

4) right and left, 5) male and female, 6) resting and moving, 7) straight and curved, 8) light and darkness, 9) good and bad, 10) square and oblong.

The correspondence among the former poles of each pair (positive pole) and the latter poles of each pair (negative pole) can be explained through various criteria. It is possible to hypothesize that some of these criteria are perceptual, since the definition of even and odd numbers is figuratively based on a particular visual structure: the gnomon (Figure 1).

The gnomon is a geometric structure used in constructing series of figure-numbers. A gnomon is the remainder of a square or rectangular figure after removing a similar figure of minor area having one corner in common. The gnomonic series show an evident geometric similarity among the figures organized in the series (geometric aspect). The same series can algebraically be described by the number of unity added to the previous figure by each increasing gnomon (algebraic aspect). As shown in Figure 1, the series of numbers relative to the square figures is the series of odd numbers; the one relative to the rectangular or oblong figure is the series of even numbers.



namely an idea of a *perceived ratio*. The possibility of directly perceiving quantity is confirmed by the results of psychological research on perceiving numerosity and on subitizing (Legrenzi, 1971; Rugelj, 1996; Miscevic, 1996; Poser, 1996).

2. The research

Hypothesis: The purpose of the research is to verify whether the opposition between “odd” and “even” has a perceptual basis in the figural structure of squared and oblong gnomons.

2.1 Method

Subjects: 33 undergraduates from the Faculty of Mathematics, 33 undergraduates from the Faculty of Philosophy.

Materials: 6 separate papers. Each paper showed

a) the Pythagorean list of opposite pairs, presented as 5 point scales of a semantic differential

b) one “target”. The targets were the two options, odd and even, for each of three variables: words “odd”, “even”; dot lattices, rectangular and squared, representation of the square and oblong Pythagorean numbers (Figure 2 a, b); margins, rectangular and squared, representations of the right angled segments composing the gnomon (Figure 2 c, d).



Figure 2 – The stimuli presented in the experiment, for the variables dot lattices, rectangular (a) and squared (b) and margins, rectangular (c) and squared (d).

Procedure: Subjects were asked to describe each target showed at the top of the paper using the scales of opposites presented below.

The 6 papers were presented in a random order. Both the order of the scales in the list and the order inside each pair (first pole-second pole) were randomized.

2.2. Results

The visual structure of even and odd

The pairs of opposites introduced by the Pythagoreans were effective in capturing the opposite characterization of the semantic descriptions of “even” and “odd” (see Figure 3). For all the scales, the profiles of the two words showed a bias to symmetry relative to the central value of the scale. For 6 of the 9 scales the difference was significant (one-plurality: $t=-4.60$; $df=65$; $p=0.000$; right-left: $t=5.48$; $df=65$; $p=0.000$; male-female: $t=-4.90$; $df=65$; $p=0.000$; light-darkness: $t=5.40$; $df=65$; $p=0.000$, good-bad: $t=5.26$; $df=65$; $p=0.000$; square-oblong: $t=9.84$; $df=65$; $p=0.000$).

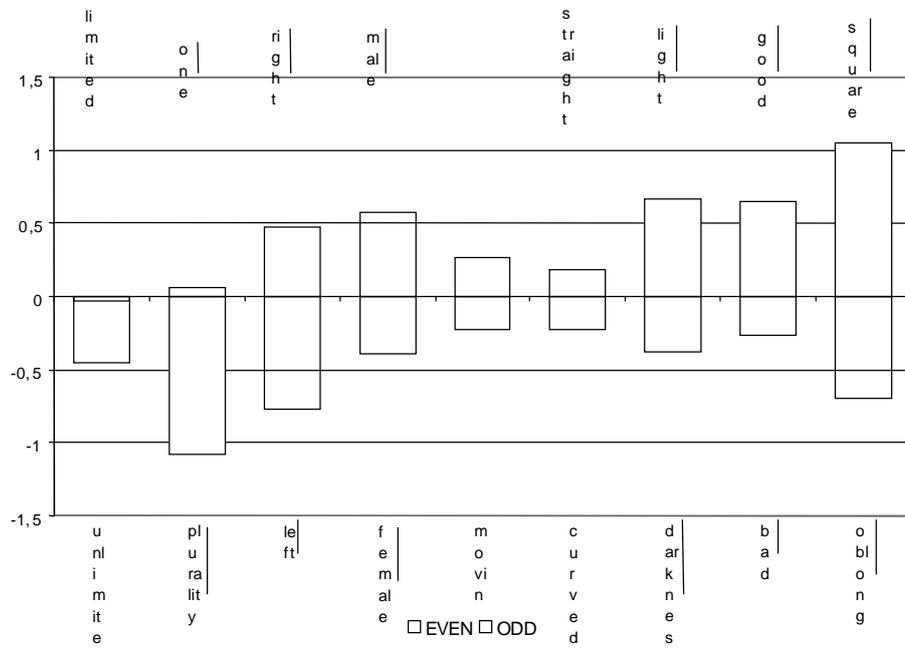


Figure 3 – Profile of the words “even” and “odd”. The two words showed a significant difference on the underlined scales.

However, the profiles of the words “even” and “odd” turned out to be inverted when compared with their Greek descriptions, revealing the influence of cultural factors for many of the qualitative attributions. This influence was

evident for the scales light-darkness, good-bad, male-female, that clearly describe cultural tendencies, being less closely related to the perceptual features of the gnomonic structure. The attribution on the scales limit-unlimited, one-plurality, resting-moving, straight-curved, were solved looking at geometric aspects of the figure-number by Pythagoreans, but not by our participants, naïve subjects unfamiliar with Pythagorean theory. Finally, the inversion of the profiles in the scale square-oblong was likely due to the fact that our participants were describing a geometric image of “even” and “odd”, instead of the algebraic description adopted as the final criteria by Pythagoreans. Therefore, in naïve phenomenology, “evenness” is associated with being square, “oddness” with being rectangular or oblong.

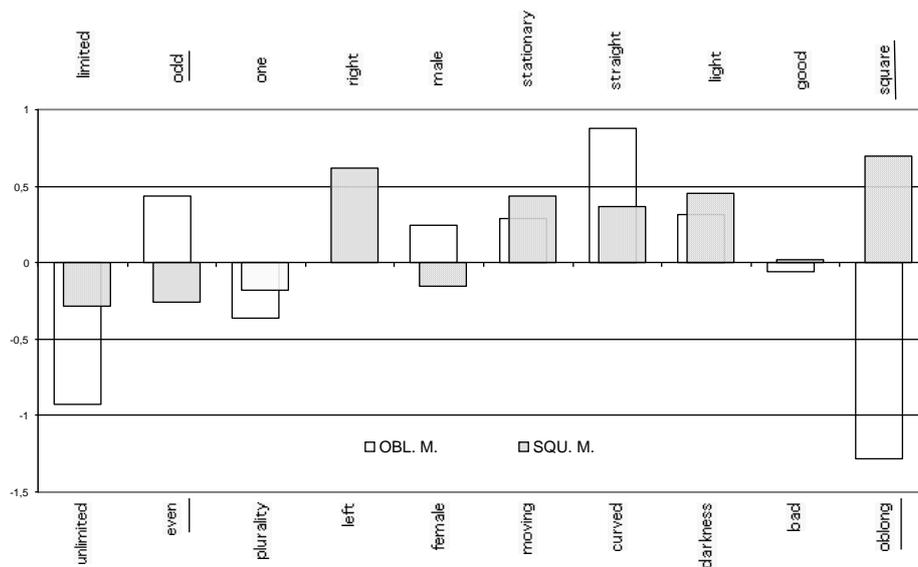


Figure 4 - Profile of the dot lattices; oblong (OBL. L.) and square (SQU. L.). The two words showed a significant difference on the underlined scales.

Looking at the values of the attributions, the square-oblong opposite turned out to be the scale that mostly reflects the definition of the two words (value >1). This result demonstrated the existence of an evident relation between the properties of these figures and the semantic definition of the opposition “even”-“odd”.

The profiles of the gnomonic structures, both dot lattices (Figure 4) and margins (Figure 5), turned out to coincide for almost all the scales, confirming that the ten Pythagorean oppositions did not directly describe geometric features of square and oblong figures. Of the obvious attributions on the scale squared-oblong, the two figures were revealed to be opposite only relative to the scale “even-odd” for the gnomonic margins ($t=3,63$; $gdl=65$; $p<0.001$), and relative to the scale one-plurality for the corresponding squared and oblong dot lattices ($t=-22.21$; $df=65$; $p=0.000$). The lattices were not discriminated on the scale even-odd, being both described as even (OBL. L. =0.76; SQU. L. = 0.33).

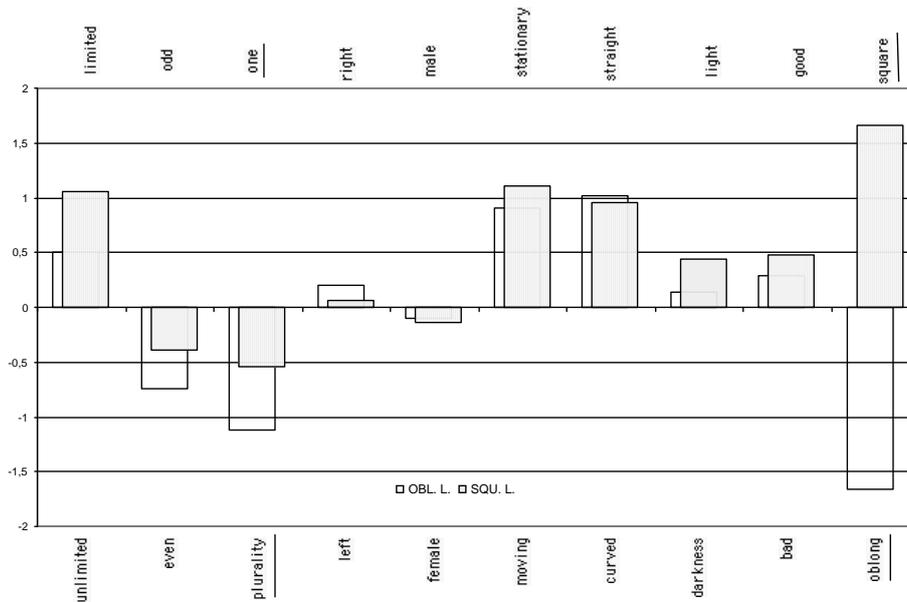


Figure 5 - Profile of the margins: oblong (OBL. M.) and square (SQU. M.). The two words showed a significant difference on the underlined scales.

3. Conclusions

This research can be considered a contribution supporting the studies of contemporary visual scientists interested in a natural history of vision (Wade,

1998). It intended to verify whether the description of the opposition between even and odd numbers has a direct reference in the visual structure of the configurations used by Pythagoreans as figure-numbers: the gnomons.

The opposition between “odd” and “even” turns out to be visible looking at the gnomonic square and oblong figures when composed only by their margins (Figure 3 c, d) and not looking at the dot lattices configurations (Figure 3 a, b).

The former configurations show a ratio between lengths, a *diastema* in the meaning of Zellini, suggesting that this is specifically the component of Pythagorean figure-numbers that could have represented a paradigmatic condition for ratio to be perceived, and for the opposition between “even” ratios and “odd” ratios to be defined.

From our perspective, this result gives further support to the claim that opposition is one of the relations directly perceivable looking at a visual configuration (Savardi, Bianchi 2000).

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