Thinking & Reasoning

ISSN: 1354-6783 (Print) 1464-0708 (Online) Journal homepage: https://www.tandfonline.com/loi/ptar20

# Overtly prompting people to “think in opposites” supports insight problem solving

Ivana Bianchi, Erika Branchini, Roberto Burro, Elena Capitani & Ugo Savardi

THINKING & REASONING
2020, VOL. 26, NO. 1, 31–67
https://doi.org/10.1080/13546783.2018.1553738



# Overtly prompting people to "think in opposites" supports insight problem solving

Ivana Bianchi[a], Erika Branchini[b], Roberto Burro[b], Elena Capitani[c] and Ugo Savardi[b]

[a]Department of Humanities, Section of Philosophy and Human Sciences, University of Macerata, Macerata, Italy; [b]Department of Human Sciences, University of Verona, Verona, Italy; [c]Department of Education, Cultural Heritage and Tourism, University of Macerata, Macerata, Italy

**ABSTRACT**
This study aims to investigate the hypothesis that "thinking in opposites" might facilitate insight problem solving. For example, if the image relating to a problem is oriented horizontally, it may be that making it vertical makes it easier to see the solution. We focus on visuo-spatial insight problem solving and study four conditions (training vs. hint, overt vs. covert identification of opposites) which differ in terms of the participants' awareness of how considering the opposites relating to a problem might lead to possible representational transformations which would help them in their search for the solution. The training condition was associated with a greater proportion of correctly solved problems. Participants who found the solution after training also made fewer attempts and did it in a shorter time. Furthermore, they referred more frequently to the instructions they had received to use opposites and made more use of the opposites they had listed in their initial exploration (overt listing condition). Overall the results show that suggesting to the participants that they "think in opposites" worked better when it was proposed as an explicit, systematic strategy.

**ARTICLE HISTORY** Received 13 August 2017; Accepted 25 November 2018

**KEYWORDS** Problem solving; visuo-spatial insight problems; opposites; hint versus training; representational change; Type 1 and Type 2 processes

## Introduction

Psychologists have used the term "insight" in many different ways when analysing the processes related to problem solving. However, something that all these interpretations have in common is that insight problems cannot simply be solved by replicating familiar rules or procedures and that a typical series of stages characterises the process underlying the search for a solution. In the initial stage, an automatic mental

**CONTACT** Erika Branchini erika.branchini@univr.it

representation of the problem arises which causes an impasse and the problem solver gets stuck. The solution then suddenly appears (the "Aha!" experience or Eureka moment) after a particular representational change is made (Ball, Marsh, Litchfield, Cook, & Booth, 2015; Danek & Wiley, 2017; Dominowski & Dallob, 1995; Fedor, Szathmáry, & Öllinger, 2015; Fleck & Weisberg, 2004, 2013; Gilhooly & Fioratou, 2009; Ohlsson, 1984a, 1984b, 1992, 2011; Öllinger & Knoblich, 2009; Perkins, 2000; Webb, Little, & Cropper, 2016).

The aim of this study is to further explore the effect that thinking in terms of opposites might have on the solution process relating to problems that require a change in representational spaces (Gilhooly, Ball, & Macchi, 2015). For example, a problem solver might think "The line I'm focusing on is horizontal … what if I orient it vertically?" or "I'm tending to stay inside the contour … what if I also try to use the space outside?" In particular, we look for new evidence regarding the effect of this prompt when suggested as an overt (explicit) strategy to be systematically applied (subsequent to specific training) or when people are primed with this suggestion as a general technique.

## *Why might opposites help?*

Evidence of a connection between divergent thinking (creativity) and the ability to manipulate opposites mentally has been found in investigations into thinking processes in various areas: from studies assessing biographical and autobiographical sources, interview studies and experimental studies concerning distinguished scientists and artists (Rothenberg, 1973a, b, 1996, 2001; Rothenberg & Sobel, 1981; Sobel & Rothenberg, 1980), to studies exploring the role of oppositional relationships in relational reasoning (e.g. Alexander, 2012, 2016; Alexander, Dumas, Grossnickle, List, & Firetto, 2016; Dumas & Alexander, 2016). There are also studies in the field of education (Antonietti, Colombo, & Pizzingrilli, 2011; Jablansky, Alexander, Dumas, & Compton, 2016; Jablansky, Alexander, & Singer, 2016; Sak, 2009; Sak & Oz, 2010).

Insight problems cannot be solved by merely applying previously learnt formulas. They require seeing the initial (obvious) representation of the problem in a new (less obvious) way. In this sense, insight processing and divergent thinking are both implied in the solution of tasks that cannot be solved by a routine search (routine problems were first defined as such by Newell & Simon, 1972). Gestalt psychologists Max Wertheimer (1919/1945) and Karl Duncker (1945) were the first to hint at the idea that *re-organising* the initial representation of a problem implies transforming some of the aspects of the initial configuration into their opposites. In Wertheimer's

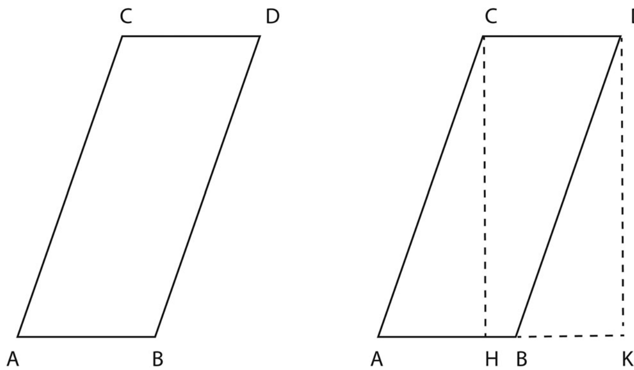
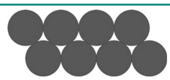
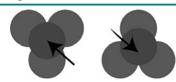
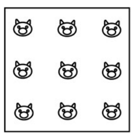
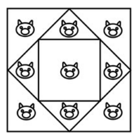
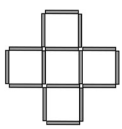
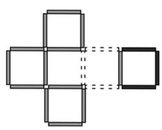
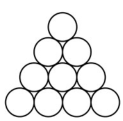
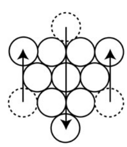
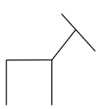
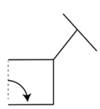



**Figure 1.** In Wertheimer's "parallelogram problem", problem solvers have to find the area of the parallelogram and demonstrate why the solution they propose is correct. The solution consists of transforming the parallelogram into a rectangle by drawing two vertical lines down from the top vertices of the parallelogram. By doing this, the problem solver can then see that the triangular area that is "missing" on the right is exactly the same size as the triangular area that "exceeds" on the left.

classic parallelogram problem (Figure 1), vertical lines drawn down from the top of a parallelogram are needed in order to see a rectangle and thus find the solution; to form the rectangle as required, a protruding area is moved from the left to the right to fill in the empty space. The dimensions which are relevant to the transformation are *vertical-horizontal*, *leaning-upright*, *left-right* and *protruding-indented*. Similarly, Duncker (1945) pointed out that the alterations that one needs to make during a productive thinking process in some cases involve shifting a function from its original state to a contrary state, and he made it clear what he meant by contraries: "For example, one calls long and short ( … ) contrary because in a pure form they are mutually exclusive. They belong to the same 'dimension', concern the object in the same respect, have the same structural locus, and consequently are in specific and active relation to one another" (Duncker, 1945, p.100).

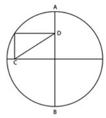
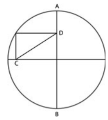
Developing this idea further, later studies have shown that prompting small groups of participants engaged in visuo-spatial problem solving to explore the problem structure in terms of opposites has the effect of inducing them to modify their approach in the search phase thus improving their success rate (Branchini, Bianchi, Burro, Capitani, & Savardi, 2016; Branchini, Burro, Bianchi, & Savardi, 2015 – examples of how the use of opposites leads to a revised assessment of the problem structure are reported in Table 1). In addition to impacting positively on the percentage of groups that succeeded in finding a solution to the problems, an analyses of the verbal interactions within the groups revealed that the problem solvers were more likely to reason with

**Table 1.** The first column lists the six problems used in the study. The second and third columns show the solution to the problem and the opposites which are critical for the solution (i.e., necessary to overcome the typical fixities).

| Problem | Solution | |
|---|---|---|
| *Figure and formulation* | *Figure* | *Critical opposites (and typical fixities)* |
|  **Eight-coin:** Participants are asked to move two coins in such a way that each coin only touches three other coins. |  | The solution requires *separating* the coins, rather than keeping them *united*; moving the *interior* coins, rather than the *exterior* ones; moving the coins downwards (3D), rather than moving them *on a* flat 2D plane. |
|  **Pigs in a pen**: Participants are asked to construct two more square pens so as to ensure that each pig ends up in a pen of its own. |  | The solution requires orienting one of the square pens so that its sides are *oblique*, rather than *horizontal* and *vertical*, as in the prototypical orientation of a square. |
|  **Five-square:** Participants are asked to reduce the number of squares from five to four by moving only three sticks. |  | The solution requires creating a configuration with a *separate* square, rather than keeping all the squares *united*; moving *interior* matches, rather than the *exterior* ones. |
|  **Triangle:** Participants are asked to make the triangle shape point downwards by moving only three circles. |  | The solution requires moving three *separate* circles, rather than three *united* circles; moving the three circles in *opposite* directions, rather than in the *same* direction. |
|  **Deer**: Participants are asked to make the deer face in a different direction by moving just one of the lines. |  | The solution requires moving the line which is on the *opposite side* of the head, i.e. the posterior leg, rather than the line forming the head; moving it from *vertical* to *horizontal*. |

(*continued*)

**Table 1.** Continued.

| Problem | Solution | |
|---|---|---|
| *Figure and formulation* | *Figure* | *Critical opposites (and typical fixities)* |
| <br>**Circumference**: Participants are asked to find the length of the oblique side CD, given the length of the diameter AB. |  | The solution requires - focusing on the *opposite* triangle (CDO), rather than the one which is more evident perceptually (CDE);seeing the initial triangle (CED) as *part* of a bigger figure (the rectangle ECOD), rather than as a *whole* figure; focusing on the EO diagonal line, rather than on the opposite diagonal (CD). |

reference to the perceptual structure of the problem and to look for perceptual solutions and perceptual operations than to refer to previously learnt notions and formulas (Branchini et al., 2015). An analysis of the drawings done by participants exposed to a training programme which showed how opposites might help them to find the solution (Branchini et al., 2016) revealed that they expanded their search space in a more focussed way and were less likely to limit their exploration of possible transformations to only one direction; they were also more inclined to abandon previously tried non-relevant properties (i.e., they manifested less fixity) and to concentrate more on properties which were in fact relevant to the solution.

The idea that manipulating the spatial properties of the initial representation of a problem in terms of their opposites might be a beneficial strategy in visuo-spatial problems (Branchini, Burro, & Savardi, 2009; Branchini et al., 2015, 2016) derives from the consideration that the direct experience of space in humans is inherently oppositional (Bianchi & Savardi, 2018; Bianchi, Savardi, & Burro, 2011; Bianchi, Savardi, & Kubovy, 2011; Savardi & Bianchi, 2009). Opposites and dimensions are key structures of conceptual spaces (Gärdenfors, 2000). Big-small, long-short, near-far, vertical-horizontal, above-below, left-right, in front of-behind, straight-bent, open-closed and so on are not only words, they correspond to primal perceptual anchor points (Bianchi, Burro, Torquati, & Savardi, 2013; Bianchi et al., 2011, 2017; on the evidence that spatial contrasts are elementary pre-linguistic schemas, see Casasola, 2008; Casasola, Cohen, & Chiarello, 2003; Quinn, Cummins, Kase, Martin, & Weissman, 1996). Transforming a given configuration or mental representation into something different implies moving within a dimension, either by modifying the degree or intensity of the property within the pole (e.g., varying the degree of curve in a curved line) or shifting the property towards the opposite pole (e.g., transforming a curved line into an angular line with varying degrees of angularity). The implications of this in terms of the processes relating to problem solving are yet to be fully understood since this is a relatively new field of experimentation. This paper aims to address one of the many questions that are still unanswered regarding to what extent opposites help the problem solver. In particular, the conditions studied in this paper differ in terms of the participants' *awareness* of how opposites might facilitate their search and in terms of how *analytical* the application of the strategy that has been suggested is. In the study, awareness and analytical application were manipulated by considering two types of facilitations, that is, training versus hints (with the former implying more awareness than the latter), and two methods to use in order to identify the oppositional dimensions implied in the initial representation of the problem (i.e., Overt Listing versus Covert Listing, with the former condition prompting a more analytical application of the "thinking in opposites" strategy).

In Gale and Ball's (2009, 2012) studies on Wason's triples, opposites were involved in an implicit hint given by the experimenters to the participants in the study. The task was to discover the rule underlying a series of triple numbers (in this case, any ascending series of number) and the hint suggested an opposite (i.e., descending series of triple numbers). In a different study (Branchini et al., 2015), the positive effect of opposites emerged in a condition where participants were asked to list all the opposites of the spatial properties relating to a problem before embarking on their search for the solution, after being told in a vague way (i.e., non-directive instructions) that this might help them. In yet another condition, thinking in opposites was proposed as an explicit strategy in Branchini et al.'s (2016) study in which participants were exposed to a brief training programme which showed how and why opposites could help them to find the solution (i.e., a directive instruction).

With regard to the debate on Type 1 and Type 2 processes (Evans & Stanovich, 2013a, b) which come into play in problem solving, the abovementioned findings on the role of thinking in opposites in hypothesis testing (Gale & Ball, 2012) and insight problem solving (Branchini et al., 2015, 2016) seem to suggest that opposites stimulate a type of reasoning which is based on both Type 1 and Type 2 processing, at different stages. In particular, as far as hypothesis testing is concerned, Gale and Ball (2012) pointed out that the solution process implies an initial phase in which problem solvers resort to a more heuristic, more automatic process which is also less effortful (i.e. Type 1). They focus on the most salient information provided in the examples and this leads to a hypothesis which is too restrictive. Later on, the initial hypothesis is revised thanks to a more systematic, analytical process (Type 2) which re-defines the boundaries of the rule the problem solvers need to find. As part of this process, adding the "descending rule" has a facilitating effect since it enlarges the boundaries of the search space thus revealing that the rule was, in fact, more general (i.e., what matters is the ascending trend, not the gap between the numbers). It might be said that this also applies to the strategy of "thinking in opposites" in insight problem solving, as exemplified in the conditions studied by Branchini et al. (2015, 2016). Problem solvers generally followed a less analytical process initially and focussed on the representation of a problem and its inherent oppositional dimensions. These were derived from the more salient information which a first assessment of the problem provided and they led to an initial hypothesis. Subsequently, by means of a more analytical process, problem solvers tended to discover the dimensions which were indeed more relevant to the solution. Evidence of this has been found in the drawings which the participants did as part of their search for a solution (Branchini et al., 2016). In this case, it was clear that the prompt to use opposites caused the problem solvers to shift their attention away from the opposites which were irrelevant to the solution. An analysis of the verbal interaction between the participants also indicated the same process (Branchini et al., 2015). Participants prompted to think in opposites tended to reconsider the constraints implied by the text of the problem and by the visual representation of the problem more frequently than the control groups (where no prompt was provided). Problem solvers focussed more frequently on figural transformations and tested them in relation to the original figural constraints of the problem. During the generation and testing of potential solutions, the prompt to think in terms of opposites facilitated a representational change by enlarging the search space while still keeping it anchored. An aspect which is still unexplored in research, however, concerns how this process can be initiated. Can problem solvers be trained to use a technique which involves an overt and systematic recourse to opposites or is it sufficient to simply hint that considering opposites will assist them?

## *Facilitation factors*

Underlying this question is a general issue regarding automatic versus aware processing and how these foster the representational changes that allow problem solvers to overcome the impasse stage (e.g., Ball et al., 2015; Bowden, Jung-Beeman, Fleck, & Kounios, 2005; Kershaw & Ohlsson, 2004; Knoblich, Ohlsson, Haider, & Rhenius, 1999; Siegler, 2000; Weisberg, 2015; Wiley & Jarosz, 2012). This topic is, in turn, part of a wider debate on the role of automatic, unconscious, associative processes (Type 1) as compared to controlled, conscious, analytical processes (Type 2) in thinking and reasoning (for an updated discussion of the dual-process theory, see Evans & Stanovich, 2013a, b). For example, some studies have shown that representational changes can benefit from changes in the distribution of memory activation, which supports the role of automatic processes (e.g., Ohlsson, 2011; Seifert, Meyer, Davidson, Patalano, & Yaniv, 1995), while others have shown the positive effects of implementing a conscious strategy of making changes or the use of heuristics (e.g., Chronicle, MacGregor, & Ormerod, 2004; Chronicle, Ormerod, & MacGregor, 2001; Kaplan & Simon, 1990).

Studying the effects of implicit or explicit hints is another way to address the issue. There is a good example of an implicit hint in Maier's (1931) study. The task (the "two cords puzzle") consisted of finding a way to tie together the ends of two cords hanging from the ceiling. The problem was that the distance between the two cords was such that if you held the end of one of the cords in your hand, the other cord was too far away to reach. The students had access to tools such as extension cords, poles, clamps and weights. When Maier saw that they were at an impasse, he "accidentally" touched one of the two cords which started swinging slightly. This was in reality a hint since the solution was to tie a weight to one of the two cords thus transforming it into a pendulum. The trick was then to hold the other cord and catch the swinging cord when it came near enough. The hint was implicit since the participants were not aware that it was in effect a suggestion and indeed most of them reported that they had not noticed it even though it was done in clear view and in effect they only managed to solve the puzzle after the hint had been given. In more recent times, the positive effect of implicit hints was found for example by Hattori, Sloman, and Orita (2013) who helped problem solvers engaged in trying to solve three classic problems (the radiation problem, the nine-dot problem and the 10-coin problem) by subliminally exposing them to images displaying the solution or by Bröderbauer, Huemer, and Riffert (2013) who exposed participants working on Katona's five-square problem to a "wave shape" represented in a logo on a sheet of paper left on a desk (but see also Gick & Holyoak, 1980; Grant & Spivey, 2003; Kershaw, Flynn, & Gordon, 2013, experiment 2).

When Gick and Holyoak (1980) tested the effect of analogical transfer, they found that presenting structurally analogous cases beforehand was not enough to improve success rates; in order for the hint to be beneficial, it needed to be overt and explicit. The same was found in Kershaw et al.'s (2013, experiment 2) study, where a positive effect of the hint emerged when participants were explicitly told to compare some problems which were analogous to the problem they had been asked to solve. Training programmes also belong to the set of explicit facilitation. They necessarily also involve hints or prompts, but always include teaching that has as its aim the development of certain skills that can subsequently be made use of (Patrick & Ahmed, 2014). The aim of training in the case of problem solving is to

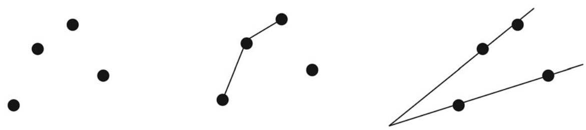


Figure 2. The figures provided during the training programme used by Dow and Mayer (2004). On the left: the figure used to present the problem. In the centre: the figure used to present the constraint removing phase. On the right: the figure presented in the conclusion phase (which shows the correct solution).

make people aware of the constraints which may cause an impasse and to suggest procedures to overcome this impasse. There are various examples of studies that have confirmed the beneficial effects of explicit training programmes such as those developed by Patrick and Ahmed (2014), Patrick, Ahmed, Smy, Seeby, and Sambrooks (2015) and Dow and Mayer (2004). To give an example, let us consider the case of Dow and Mayer's (2004) training for spatial insight problems (pp. 394–395). It consisted of first advising participants that the solution could be reached by "ignoring a self-imposed constraint (or block) rather than trying various actions" followed by an explanation of a three-step procedure. The example they used was the line problem: "Without lifting your pencil from the paper, show how you could join all 4 dots in the diagram with two continuous straight lines" (Figure 2). In the first step (i.e., finding the constraint), the participants were told to ask themselves a question: "Do I need to stop at each dot or can I draw through it and keep going?" They were then shown a picture with a line drawn from one dot to another and then another. In the second step (i.e., removing the constraint), participants were invited to: "Draw a line through the dots." In the third step (i.e., the conclusion), there was a picture showing one line going through two of the dots and another line through the other two dots with the lines meeting at a point outside the dots. It was also explained to the participants that a common mistake is to stop at each dot.

In our study, we provided problem solvers with either training or a hint. Before presenting the details of the study, it is important to note that some of the hints and training programmes mentioned thus far operated mainly on a linguistic level (e.g., Gibson, Dhuse, Hrachovec, & Grimm, 2011; Patrick & Ahmed, 2014; Patrick et al., 2015; Walinga, Cunningham, & MacGregor, 2011), while others aimed to stimulate visuo-spatial elaboration, either directly (e.g., Bröderbauer et al., 2013; Grant & Spivey, 2003; Hattori et al., 2013) or by weakening the possibility of engaging in speech-based processing (e.g., Ball et al., 2015; Schooler, Ohlsson, & Brooks, 1993). The conditions in our study were, in a way, a combination of the above: on the one hand, participants were asked to verbally identify pairs of opposite properties while at the same time this identification was based on a visual analysis of the spatial structure of the problem.

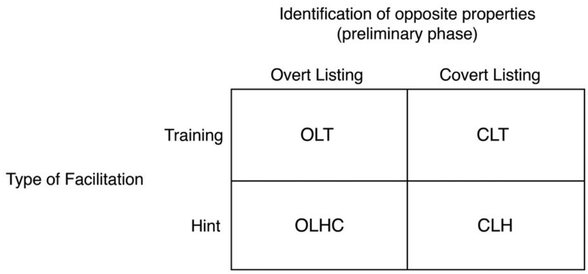



**Figure 3.** Diagram representing the two factors which the four experimental conditions were based on.

## *The study*

The study aimed to explore the role of different degrees of awareness of how opposites can be used to support representational changes in visuo-spatial insight problem solving. The experimental design of the study allowed a direct comparison between the various conditions presupposing different levels of awareness of the use of opposites in problem solving and different levels of systematicity in the application of the strategy. We investigated the effects of two factors and of their interaction (Figure 3).

With regards to the first factor, *Type of Facilitation*, we compared the effects of a *hint* and a *training* programme. In the hint condition, the participants were advised that there would be a preliminary task which would require them to identify all the spatial properties characterising the problem and their corresponding opposites and that this would help them to find the solution. In the training condition, it was explained to them in detail that identifying opposites might help and a strategy was then suggested and exemplified. The hypothesis was that if the groups exposed to training performed better than the groups in the hint condition, then this would indicate that awareness of the way in which using opposites might help is a critical feature. Conversely, a better performance in the hint condition would suggest that the positive impact of a prompt to think in terms of opposites is not necessarily driven by a focussed, analytical and systematic approach.

The second factor, *Identification* of *opposite properties*, consisted of two variants of the initial step which involved identifying all the spatial properties characterising the problem and their corresponding opposites. This was done in both the hint and training conditions. In one case (Overt Listing), the participants were asked not only to look for the opposites, but also to write them down. Overt listing was expected to favour a more analytical application of the use of opposites, with the idea being that the list would systematically drive the search phase. In the other condition, the participants were not asked to actually write down the list (Covert Listing), and the suggestion to use opposites was expected to work as a primer rather than a strategy to be analytically applied.

Further information regarding how participants assimilated the prompt was expected to emerge from a comparison between the opposites identified in the instructions given by the experimenter or listed by the participants in the preliminary phase (in the Overt Listing condition) and those which were in effect manipulated in the search phase. A marked overlap between the former and the latter would suggest that the participants were using the initial pairs to drive their exploration in the search phase and this is more compatible with a focussed, systematic and analytical approach. Conversely, if the overlap was small and the participants tended to manipulate other properties than those suggested in the instructions or listed in the preliminary phase, this would suggest that the initial listing merely primed them in a certain direction rather than actually suggesting the "contents" to be tested.

## Method

### Participants

Two hundred and forty undergraduate students (111 males, 129 females; $M_{\text{age}}$ = 21.83 years, *SD* = 7.26 years) took part in the study in small groups of three. A number of researchers have suggested that three is the ideal number of group members in order to obtain an accurate description of the properties characterising an event under observation and to improve performance in complex problem-solving tasks (e.g. Bozzi, 1978/2019; Brophy, 2006; Laughlin, Bonner, & Miner, 2002; Laughlin & Futoran, 1985; Moshman & Geil, 1998). Sixty participants (i.e., 20 groups) were randomly assigned to each of the four conditions (Figure 3). They all gave their informed consent prior to the beginning the experiment. The study conforms to the ethical principles of the declaration of Helsinki (World Medical Association, 2013) and was approved by the ethical committees of the University Departments of the researchers involved in the study.

The decision to stop data collection after 20 groups was based on the following consideration. Given the decision to report only significant results (i.e., with a Cohen's *d* greater than "small", typically 0.2), the minimum number for a Chi-square with $d = 0.2$, df $= 1$, sig. level $= 0.05$ and power $= 0.8$ equals 196 corresponded, in fact, to the number of cases we obtained. It is important to bear in mind that it is the number of attempts (not groups) that constitutes a statistical unit in all of the following analyses.

## Experimental design

The four conditions represented in Figure 3 correspond to a $2 \times 2$ design. One factor was Type of Facilitation (Hint, Training) and another factor was Identification of opposite properties (Overt, Covert). The four resulting conditions were studied between participants: Overt Listing Training (OLT), Covert Listing Training (CLT), Overt Listing Hint (OLH) and Covert Listing Hint (CLH).

In the Training conditions, the participants were requested to work in order to: (1) identify all the spatial properties pertaining to the problem; (2) identify the opposites of each of the properties they had found and (3) ascertain whether transforming the representation of the problem by means of these opposites would help them to find the solution. To exemplify the procedure, they were shown three problems (see Appendix 1).

In the Overt Listing variant of the Training condition (OLT), the participants were asked in the first step to list the spatial properties on a piece of paper, and in step 2 to list the corresponding opposites alongside on the same piece of paper. In the Covert Listing Training condition (CLT), they were asked to follow the same three steps but they were not required to make lists.

In the Hint condition, the participants were told to identify all the spatial features which were mentioned in the text of the problem or were visible in the corresponding figure and then find their opposites. They were advised that this strategy would help them to find the correct solution. To exemplify the procedure, the participants were shown three example problems and their relative spatial features with their corresponding opposites (the same as those used in the training condition) (see Appendix 1). As in the training condition, in the Overt Listing variant of the Hint condition (OLH), the participants were asked to write down a list of spatial properties they found and their corresponding opposites, whereas in the Covert Listing variant of the Hint condition (CLH) this was not one of the required steps.

## Materials

Six visuo-spatial problems were used in the test phase (Table 1): the "pigs in a pen" problem (Schooler et al., 1993); the "triangle" problem (e.g., De Bono, 1969); the "deer" problem (origin unknown); the "eight-coin" problem (Ormerod et al., 2002); the "five-square" problem (Katona, 1940) and the "circumference" problem (Köhler, 1969). The order of these problems was randomised between participants.

The three example problems used in the instructions to the participants were the "parallelogram problem" and the "altar-window problem" (both devised by Wertheimer, 1919/1945) and the "nine-dot problem" (devised by Maier, 1930) – see Appendix 1. They were also given the answers to these problems (i.e., they were used as descriptive examples).

### Procedure

The study was carried out in a room at the University of Verona, with one group at a time. The participants were recruited from the University and volunteered to take part in the study.

The experimenter read aloud the text with the instructions concerning the training session or hint (while also projecting these on a screen). During and at the end of the explanations, participants had the opportunity to ask the experimenter for clarification if necessary. Once it was clear that the instructions had been understood, the experimenter presented the participants with one booklet per group containing the problems to be solved. The six problems were printed on separate A4 sheets of paper, with the order randomised between groups. Only one booklet per group was provided in order to stimulate the participants to work together. In all of the conditions, it was emphasised to them that they had to work together, talking aloud and sharing their thoughts. They were given seven and a half minutes for each problem and were instructed to raise their hands when they thought they had found the correct solution. If the solution was correct, the response time was recorded, if not, they were encouraged to keep searching until the end of the time at their disposal.

All the sessions were video-recorded (with the written consent of the participants) in order for there to be audio-visual support for the experimenters when they classified the responses. This was done to avoid potential uncertainties in the interpretation of the drawings participants sketched while searching for a solution.

### Procedure followed in the classification of responses

We decided to consider each drawing done in the search phase as an attempt to find a solution (as in Branchini et al., 2016). The reason for focussing on the drawings rather on the verbal dialogues between the group members is that they can be regarded as behavioural correlates of the cognitive search space that the participants are exploring and as such reveal their aware and unaware cognitive processes (Fedor et al., 2015). We also felt that focussing uniquely on the instances when the participants announced that they had found a solution by raising their hands was too restrictive since the productive, "figuring out" phase of the process was, for our purposes, much richer.

Two additional analyses were conducted in the Overt Listing conditions (Training vs. Hints and OLT vs. OLH, respectively) in order to study the categories pertaining to the opposites listed (see Analysis of the lists produced by partipants in the OLT and OLH conditions: did difference emerge in terms of the quantity and quality (i.e., content and novelty) of the opposite listed? in the Results section) and the dimensions which had been transformed in the drawings (see The relationship beyween the opposites listed and those manipulated by the particpants in the drawings done in the search phase in the Results section). These analyses required qualitative classifications which were carried out by two independent judges who were among the authors of the present paper.

The categories relating to the contents of the lists of opposites (Analysis of the lists produced by partipants in the OLT and OLH conditions: did difference emerge in terms of the quantity and quality (i.e., content and novelty) of the opposite listed? in the Result section) were inductively defined by all of the authors after an initial inspection, the premise being that the task asked participants to find as many *spatial* opposites as possible. The following categories were identified: direct Spatial Opposites (dSO), e.g., *horizontal-vertical*; indirect Spatial Opposites (iSO), e.g., *square-circle* (note that the term *indirect* is used to indicate that critical opposite properties, i.e., *angular-rounded*, were not directly mentioned but rather suggested when the objects which manifested these properties were referred to verbally); non-Spatial Opposites (nSO), e.g., *light-dark* in colour and lastly oppositions expressed by NEGation (NEG), e.g., *oblique-not oblique*. There was a further category to encapsulate non Opposites (nO), i.e., all cases of incorrect or incomplete responses such as when a characteristic of the problem was referred to but it was not paired with its opposite, for instance, “It’s an *equilateral* triangle”. The classification was established by two independent judges based on the lists which the participants had written down. This was supported by the video recordings of the experimental session which enabled us to assess the completeness of the lists and clarify the meaning of any pairs which were unclear.

The other dependent variable which we analysed and classified qualitatively concerned the relationship between the opposites listed in the preliminary phase and those manipulated by the participants during the search phase in the form of drawings (The relationship beyween the opposites listed and those manipulated by the particpants in the drawings done in the search phase in the Results section). Each drawing was classified in terms of whether or not the transformation of the initial representation of the problem displayed in the drawing corresponded to the pairs listed in the preliminary phase. For example, in the deer problem, if the pair “*vertical-horizontal*” was listed and then in the drawing the vertical leg of the deer was removed and positioned horizontally, this was marked as a transformation relating to that dimension, that is to say, there was a correspondence (which the participants were either aware or unaware of) between the pair previously listed and the transformation depicted in the drawing. Conversely, if the drawing showed, for example, that the straight line corresponding to the head of the deer had been transformed into a curved line, this was classified as an attempt which had *no* correspondence to the listed pair (i.e., vertical-horizontal). The classification was made by two independent judges for each pair of opposites listed by each group, after having excluded incomplete pairs (i.e., nO, as defined above).

## Data analysis

Generalised Mixed Effect Models (GLMM) were used to analyse the data. By means of these models it was possible to deal with the variability related to the Problems and the Subjects as random effects while considering the two experimental conditions (Type of Facilitation and Identification of opposite properties) as fixed effects. Random effects have factor levels that do not exhaust the possibilities. If one of the levels of a variable were replaced by another level, the study would be essentially unchanged (Borenstein, Hedges, Higgins, & Rothstein, 2009). For the purposes of the hypotheses tested in our study, the problems used in the experiment were simply exemplars of a general category (i.e. visuo-spatial geometrical problems) of varying degrees of difficulty (see Branchini et al., 2016, Table 2). For this reason, we entered Problems in the model as a random factor.

All analyses were carried out using the statistical software program R 3.3.1, with the “lme4” (Bates, Machler, Bolker, & Walker, 2015), “car” (Fox & Weisberg, 2011), “lsmeans” (Lenth, 2016), and “effects” (Fox, 2003) packages. We performed Mixed Model ANOVA Tables (Type 3 tests) via likelihood ratio tests (Barr, 2013; Barr, Levy, Scheepers, & Tily, 2013; Bates, Kliegl, Vasishth, & Baayen, 2015) implemented in the “afex” package (Singmann, Bolker, & Westfall, 2017). Bonferroni corrections were applied to post-hoc comparisons.

## Results

The results of the study will be discussed in relation to each of the dependent variables analysed and the research questions that the study aimed to answer in terms of: (a) success rate, that is, the proportion of correctly solved problems; (b) the number of attempts made; (c) the time taken to reach a solution; (d) the quantity and quality (i.e., type and novelty) of the opposite pairs listed and (e) the relationship between the opposite properties listed and those which were transformed in the drawings.

## *Success rate*

Did the number of problems solved vary in relation to the Type of Facilitation provided (Hint or Training) and/or the Overt or Covert Identification of the opposite properties in the preliminary phase?

A GLMM was conducted on Response (on two levels: solution found, not found) with Type of Facilitation and Identification of opposite properties as fixed effects. A main effect of Type of Facilitation emerged ($\chi 2_{(1,\ N\ =\ 80)}$ = 6.530, $p = .011$, $z$-ratio = −3.993, $d$ = −0.446), which confirmed that the participants in the Training condition solved more problems than the participants in the Hint condition, whereas only a trend emerged for Identification of opposite properties ($\chi 2_{(1,\ N=80)}$ = 3.123, $p = .077$, $z$-ratio =1.981, $d = 0.221$), indicating that an Overt Listing of the spatial features characterising the structure of the problem (and their opposites) was associated with only a tendency to be more successful. The significant interaction between the two factors ($\chi 2_{(1,\ N=80)}$ = 4.453, $p = .035$; see Figure 4) made it clear that, in effect, producing an Overt Listing only led to a better performance in the Hint condition (Bonferroni post-hoc: OLH vs. CLH: $EST = 0.986$, $SE = 0.354$, $z$-ratio = 2.781, $p = 0.034$, $d = 0.311$), whereas no differences emerged between Overt and Covert Listing in the two Training conditions (OLT vs. CLT: $EST$ = −0.0524, $SE = 0.397$, $z$-ratio = −0.131, $p = 1.000$). The post-hoc tests also revealed that the success rate in the Covert Listing Hint condition (CLH) was significantly worse than in all of the other three conditions (CLH vs. CLT: $EST$ = −1.128, $SE = 0.416$, $z$-ratio = −3.073,

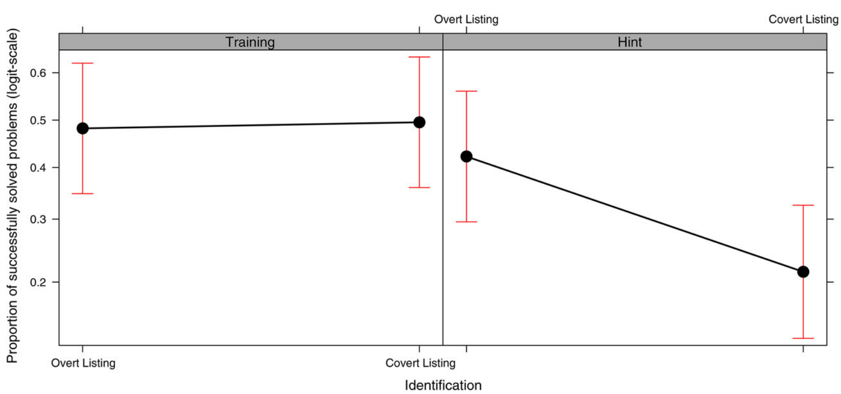



**Figure 4.** Effect plot of the interaction between the two Types of Facilitation (Training and Hint) and Identification of opposite properties in the initial phase (Overt Listing and Covert Listing) in terms of the proportion of successfully solved problems (on a logit scale). The bars represent the 95% confidence interval.

$p$ = .012, $d$ = −0.344; CLH vs. OLT: *EST* = −1.227, *SE* = 0.415, $z$-ratio =2.957, $p$ = .018, $d$ = −0.331). The latter three conditions (i.e., OLH, OLT, CLT) had similar success rates (OLH vs. OLT: *EST* = −0.240, *SE* = 0.399, $z$-ratio = −0.602, $p$ = 1.000; OLH vs. CLT: EST = −0.293, *SE* = 0.400, $z$-ratio = −0.731, $p$ = 1.000).

## *Number of attempts (manifested by the number of drawings produced in the search phase)*

A GLMM (Poisson family, log link function) was carried out on the drawings done by each group in order to test whether the number of attempts made by the groups in the search phase varied in relation to the Type of Facilitation provided (Hint, Training), the initial Identification of the problem's properties (Overt Listing, Covert Listing), and the success rate (Solution found, not found).

No main effect of Identification of opposite properties emerged ($\chi2_{(1,\ N\ =\ 80)}$ = 0.067, $p$ = .795). This indicates that making an Overt List of opposite properties in the preliminary phase did not lead *per se* to a difference in the number of drawings done in the time at their disposal.

A significant main effect of Response emerged, as shown in the t graph on the left in Figure 5 ($\chi2_{(1,\ N\ =\ 80)}$ = 40.721, $p < .001$, $d = 0.734$): a higher number of attempts was found to be associated with an inability to solve the problems. This reason for this was that the participants who could not come up with a solution kept trying and did more and more drawings until the end of the time limit, while those who found the correct solution stopped. The interaction between Type of Facilitation and Response ($\chi2_{(1,\ N\ =\ 80)}$ = 10.235, $p$ = .001) revealed that the training did not lead in general to fewer attempts: this applied specifically when those participants who had undergone training subsequently succeeded in solving the problems (solvedT vs. solvedH: *EST* = −0.275, *SE* = 0.098, $z$-ratio = −2.788,

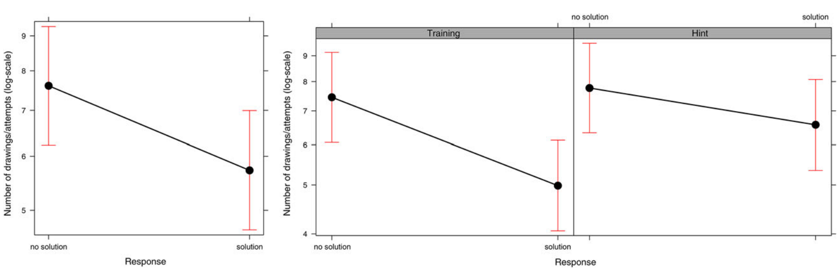



**Figure 5.** Effect plots showing how the number of drawings done/attempts made in the solution search phase varied depending on whether the participants succeeded or not in finding the solution (left graph), and on the interaction with Type of Facilitation (right graph). All plots are on a log-scale. The bars represent the 95% confidence interval.

$p = .0317$, $d = 0.312$; solvedT vs. not-solvedH: $EST = -0.443$, $SE = 0.095$, $z$-ratio $= -4.634$, $p < .001$, $d = -0.518$; not-solvedT vs. solvedH: $EST = 0.125$, $SE = 0.098$, $z$-ratio $= 1.273$, $p = 1.000$; not-solvedT vs. not-solvedH: $EST = -0.042$, $SE = 0.089$, $z$-ratio $= -0.471$, $p = 1.000$).

## Time required to find the solution

The finding that the participants exposed to training who then succeeded in finding the solution made fewer attempts than those who found the solution in the hint condition (see previous section) raises the question of whether the former were also faster. We wondered whether being exposed to training might have led participants to be more reflective about their course of action and therefore less rushed about making and abandoning various attempts, thus resulting in a smaller number of attempts made but over the same or a longer period of time. Conversely, if a smaller number of attempts was associated with shorter solution times, this would be an indication that training had the effect of guiding the participants towards the solution more quickly.

A Linear Mixed Effect Model was carried out on the time taken to find the solution (for the groups who succeeded in solving the problems), with Type of Facilitation and Identification of opposite properties as fixed effects. The analysis was performed after having checked for the normality of distribution of the data (using qqnorm function and shapiro-test in R's stats-package). The main effect of both Identification of opposite properties and Type of Facilitation turned out to be significant (Figure 6, top panels) whereas the interaction between the two factors was non-significant ($\chi2_{(1,\ N = 80)} = 0.947$, $p = .330$). The solutions found after an Overt Listing of opposite properties were generally associated with longer response times as compared to the those found in the Covert Listing condition ($\chi2_{(1,\ N = 80)} = 4.441$, $p = .035$; $z$-ratio $= 1.996$, $d = 0.223$). The Training condition was in general associated with faster solutions than the Hint condition (main effect of Type of Facilitation: $\chi2_{(1,\ N = 80)} = 4.944$, $p = .026$, $z$-ratio $= 2.105$, $d = 0.235$).

Since survival analysis methods can reveal useful information about the time taken to generate potential solutions in insight problem solving (e.g., Litchfield & Ball, 2011), the proportion of problems solved correctly over time and across conditions was also examined by means of this technique (Survival Package in R version 2.42-3, https://cran.r-project.org/web/packages/survival/index.html; Therneau & Grambsch, 2000) using the non-parametric Kaplan Meier method (Harrington & Fleming, 1982; Kaplan & Meier, 1958). To compare survival functions statistically, we used the Mantel Cox log-rank test (e.g., see Collett, 2003), which is a goodness-of-fit test (in this case a Chi-square test). As shown in the

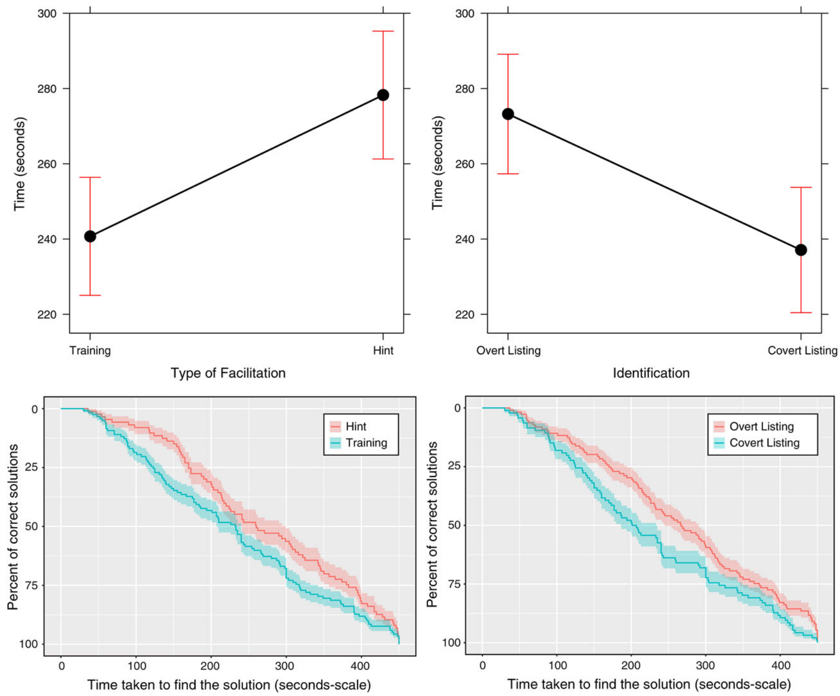



**Figure 6.** Top panels: Fixed effect plots relating to the time taken to find the solution (seconds-scale). Main effect of Type of Facilitation, Training or Hint (graph on the left) main effect of Identification of the opposite properties, Overt or Covert Listing (graph on the right). Bottom panels: Survival analysis curves representing a comparison of the proportions of problems solved across conditions over time. In all graphs, the bars represent a 95% confidence interval.

bottom panels in Figure 6 (also confirmed by the chi-square test on the two curves), the two curves differ both for Type of Facilitation, with a better performance in the Training condition (see bottom left panel in Figure 6; $\chi 2_{(1,\ N\ =\ 205)} = 3.999$, $p = .045$) and Identification of opposite properties, with a better performance for the Covert Listing condition (see bottom right panel in Figure 6; $\chi 2_{(1,\ N\ =\ 205)} = 6.467$, $p = .011$).

Further analyses were carried out focusing only on the Overt Listing conditions (OLT and OLH). We looked into the number of different opposites listed and their quality in the two conditions and into the relationship between the opposites listed in this preliminary phase and the transformations explored by the participants in the subsequent search phase (identified by means of the drawings they did). We also explored how frequently explicit references to the strategy involving the use of opposites were made during the search phase in order to assess any intention to drive the process along these lines.

### Analysis of the lists produced by participants in the OLT and OLH conditions: did differences emerge in terms of the quantity and quality (i.e., content and novelty) of the opposites listed?

### Length of the lists

We first of all wondered whether the two Types of Instructions (Training or Hint) influenced the *number* of opposite properties identified and listed by participants. A GLMM (Poisson family with Type of Facilitation as fixed effect, and Groups and Problems as random effects) was conducted and, as shown in Figure 7 (top left graph), the participants in the Hint condition produced longer lists than those in the Training condition ($\chi2_{(1,\ N\ =\ 40)}$ = 33.431, $p < .001$, $z$-ratio = 5.782, $d = 0.914$).

### Quality (content) of the pairs of opposites

Two independent judges classified each opposite pair into five different categories (direct Spatial Opposites, dSO, e.g. *horizontal-vertical*; indirect Spatial Opposites, iSO, e.g. *square-circle*; non-Spatial Opposites, nSO, e.g., *light-dark* in colour; NEGation, NEG, e.g. *oblique-not oblique*.; non Opposites,

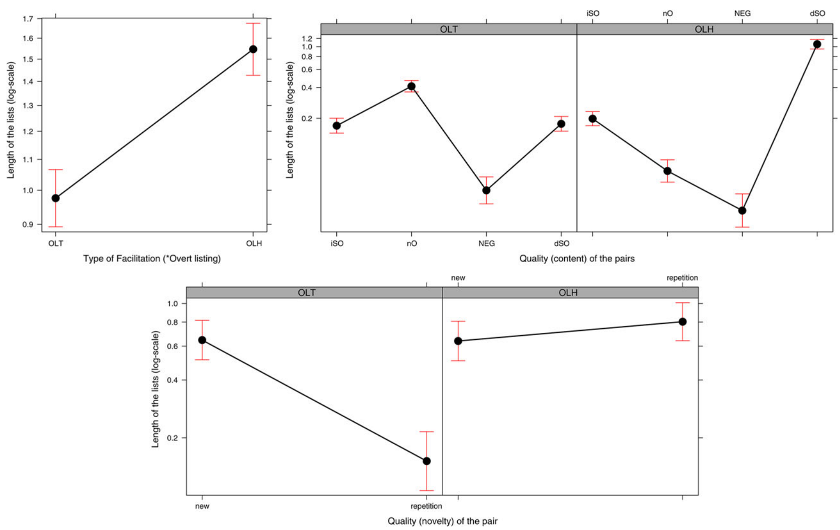



Figure 7. Fixed effect plots relating to the GLMM analyses of various characteristics of the lists produced in the preliminary phase in the Overt Listing Training (OLT) and Overt Listing Hint (OLH) conditions. Top left graph: the number of opposites listed for all six problems. Top right graph: the quality of the pairs listed, i.e., direct Spatial Opposites (dSO), indirect Spatial Opposites (iSO), NEGations (NEG) and non Opposites (nO). Bottom graph: the quality of the pairs listed in terms of novelty, i.e., new pairs or a repetition of those mentioned in the instructions. The bars represent a 95% confidence interval.

**Table 2.** Bonferroni post-hoc tests concerning the frequency of the various Quality categories relating to the pairs of opposites listed by participants (direct Spatial Opposites (dSO), indirect Spatial Opposites (iSO), NEGations (NEG) and non Opposites (nO) in the Overt Listing Hint (OLH) and Overt Listing Training (OLT) conditions.

| | Contrasts | Estimates | SE | *z*-ratio | *p*-Value | *d* (Cohen) |
|---|---|---|---|---|---|---|
| a | OLH_NEG vs. OLT_NEG | −0.4519 | 0.4780 | −0.9454 | 1.0000 | |
| b | OLH_iSO vs. OLT_iSO | 0.1571 | 0.1964 | 0.8003 | 1.0000 | |
| c | OLH_dSO vs. OLT_dSO | 1.7883 | 0.1526 | 11.7185 | <0.0001 | 1.853 |
| d | OLH_nO vs. OLT_nO | −1.9029 | 0.2570 | −7.4025 | <0.0001 | −1.170 |
| e | OLH_dSO vs. OLH_iSO | 1.6728 | 0.1453 | 11.5128 | <0.0001 | 1.820 |
| f | OLH,iSO vs. OLH_nO | 1.1741 | 0.2743 | 4.2791 | 0.0005 | 0.676 |
| g | OLH_nO vs. OLH_NEG | 0.8873 | 0.4440 | 1.9982 | 1.0000 | |
| h | OLT,nO vs. OLT_dSO | 0.8443 | 0.1689 | 4.9991 | <0.0001 | 0.790 |
| i | OLT,nO vs. OLT_iSO | 0.8860 | 0.1713 | 5.1695 | <0.0001 | 0.817 |
| l | OLT_iSO vs. OLT_dSO | 0.0416 | 0.2018 | 0.2064 | 1.000 | |
| m | OLT_dSO vs. OLT_NEG | 1.4939 | 0.3298 | 4.5284 | 0.0002 | 0.716 |
| n | OLT_iSO vs. OLT_NEG | 1.4522 | 0.3311 | 4.3851 | 0.0003 | 0.693 |

nO). The final inter-rater agreement was good (*Cohen's* $\kappa = 0.881$, $SE = 0.043$). A GLMM was carried out to study the frequency of the various categories in the OLT and OLH conditions (Poisson family with Type of Facilitation and Quality of the pairs as fixed effects and Groups and Problems as random effects). Since non Spatial Opposites (nSO) were very infrequent (they represented 4% of the total number of opposites listed), we eliminated them and ran the analyses on the other four more frequent categories. The results are displayed in the top right graph in Figure 7 which shows the significant interaction between Type of Facilitation and Quality of the pairs ($\chi 2_{\ (3,\ N\ =\ 40)}$ = 165.263, $p < .001$). As shown in the graphs and confirmed by post-hoc tests (see Table 2, lines a–d), the quality of the opposites listed differed in the two conditions (OLH and OLT). These differences did not concern the frequency of NEGations (NEG) or of indirect Spatial Opposites (iSO) but the frequency of direct Spatial Opposites (dSO) and incorrect/incomplete pairs (non Opposites, nO). Direct Spatial Opposites (dSO) were more frequent and nonOpposites (nO) were less frequent in the lists produced in the OLH condition than in the OLT condition.

This difference between the two conditions also emerges if we look at the relative frequency of each category within each of the two conditions. In the OLH condition, direct Spatial Opposites (dSO) were significantly more frequent than indirect Spatial Opposites (iSO), which were more frequent than non Opposites (nO) which, in turn, were as frequent as NEGations (NEG) – see post-hoc tests e–g in Table 2. Conversely, in the OLT condition, non Opposites (nO) were more frequent than direct Spatial Opposites (dSO) and indirect Spatial Opposites (iSO) – with no difference between the two – both of which were more frequent than NEGations (NEG); see post-hoc tests h–n in Table 2.

## *Quality (novelty) of the pairs of opposites*

We also analysed the novelty of the pairs in the lists. Each of the opposites listed by the participants was categorised as either "new" or "repetition" depending on whether it was, respectively, a new pair of opposites identified beyond those used in the instructions or a pair which had already been mentioned in the instructions in order to explain the task (*Cohen's* $\kappa = 0.956$, $SE = 0.021$). A GLMM was performed on the frequency of "new" versus "repeated" pairs, in the Overt Training and Overt Hint conditions (Poisson family with Type of Facilitation and Novelty of the pair as fixed effects and Groups and Problems as random effects). The result of the analysis is shown in the bottom graph in Figure 7, which shows the interaction between Novelty and Type of Facilitation ($\chi2_{(1,\ N\ =\ 40)} = 72.099$, $p < .001$). Whereas the lists produced in the Hint condition contained a similar number of new and replicated opposites (OLH_rep vs. OLH_new: $EST = -0.231$, $SE = 0.100$, $z$-ratio $= -2.305$, $p = .127$), in the Training condition, participants listed new opposites more frequently (OLT_rep vs. OLT_new: EST $= -1.449$, $SE = 0.170$, $z$-ratio $= -8.493$, $p < .001$, $d = 1.343$).

## *The relationship between the opposites listed and those manipulated by the participants in the drawings done in the search phase*

For each pair of opposites listed by each group, two independent judges classified the pairs according to whether they corresponded or not to the transformations displayed in each individual drawing (see Procedure section). The inter-rater agreement between the two judges was good (*Cohen's* $\kappa = 0.875$, $SE = 0.052$). A first GLMM was conducted on the proportion of pairs listed in the preliminary phase and subsequently used in the drawings (Binomial family, with Type of Facilitation as fixed effect and Groups and Problems as random effects.). As shown in Figure 8 (top graph), the participants tended to use the opposites mentioned in the preliminary phase more often in the OLT than in the OLH condition ($\chi2_{(1,\ N\ =\ 35)} = 28.990$, $p < .001$, $z$-ratio: $-5.384$, $d = -0.910$).

A second GLMM (Poisson family with Type of Facilitation and Quality as fixed effect and Groups and Problems as random effects) was conducted to study whether the relationship between the opposites listed and those subsequently used in the drawings was independent of the Quality of the pairs. The analysis was carried out on the frequency of the opposites listed and then used in the drawings in relation to the three different Quality classes (direct Spatial Opposites, indirect Spatial Opposites or NEGation). This was done after having excluded the incorrect category, i.e. non Opposite (nO). A main effect of Quality of the pair emerged ($\chi2_{(2,\ N\ =\ 35)} = 41.696$, $p < .001$): the listed opposites which were more frequently used in the drawings

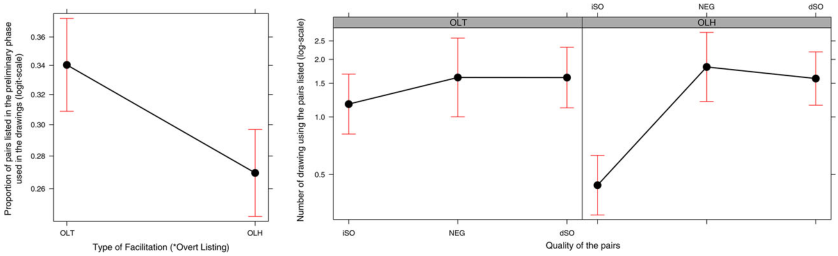



**Figure 8.** Fixed effect plots relating to the proportion of pairs listed by participants in the preliminary phase which corresponded to those subsequently used and/or transformed in the drawings done in the search phase. Data are presented separately for the two Facilitation conditions (graph on the left) and in interaction with the quality of the pair (direct Spatial Opposites, dSO; NEGation, NEG; indirect Spatial Opposites, iSO). Bars represent a 95% confidence interval.

belonged to the category of direct Spatial Opposites and NEGation (NEG vs. iSO: $EST = 0.8254$, $SE = 0.240$, $z$-ratio $= 3.425$, $p = 0.002$, $d = 0.579$; dSO vs. iSO: $EST = 0.843$, $SE = 0.128$, $z$-ratio $= 6.566$, $p < .001$, $d = 1.110$). A significant interaction between Quality of opposites and Type of Facilitation ($\chi 2_{(2,\ N\ =\ 35)} = 12.634$, $p = .001$), however, revealed that indirect Spatial Opposites (see Figure 8, graph on the right) were also frequently used in the Training condition, but not in the Hint condition (OLT_iSO vs. OLH_iSO: $EST = 0.876$, $SE = 0.219$, $z$-ratio: 3.992, $p = .001$, $d = 0.674$).

### *Overt references (verbally expressed by participants in the search phase) to the strategy which had been suggested or to the examples given in the explanation of the task*

We used the video recordings of the experimental sessions in order to count the number of times that the participants verbally reminded themselves and/or the other members of the group to apply the strategy they had been prompted to use. A GLMM was then performed on the frequency of these explicit references to the strategy (Poisson with Type of Facilitation as fixed effect and Groups and Problems as random effects). As the significant effect of Type of Facilitation revealed ($\chi 2_{(1,\ N\ =\ 40)} = 3.932$, $p = .047$, $z$-ratio $= 1.992$, $d = 0.315$), explicit references to the strategy were more frequent in the Training (OLT) as compared to the Hint (OLH) condition (Figure 9, graph on the left).

We also analysed how frequently the participants re-examined the examples given to them to explain the strategy by counting the number of times they verbally referred to them. The GLMM (Poisson with Type of Facilitation as fixed effect and Groups and Problems as random effects) performed on these data revealed a more frequent explicit reference to the examples in

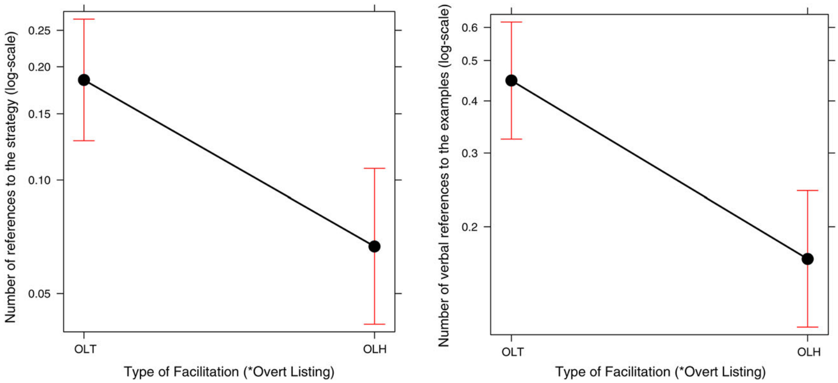



**Figure 9.** Effect plots of the two Types of Facilitation in the Overt Listing condition (Hint, OLH, and Training, OLT) on the frequency of verbal reminders of the strategy to be applied (graph on the left) and of the examples used in the instruction phase (graph on the right) taken from the dialogues of the participants in the search phase. Bars represent a 95% confidence interval.

the Training (OLT) as compared to the Hint (OLH) condition ($\chi 2_{(1,\ N\ =\ 40)}$ = 5.498, $p = .019$, $z$-ratio =2.345, $d = 0.370$; Figure 9, graph on the right).

## Final discussion

In this study, we compared the effects on visuo-spatial insight problem solving of explicit or implicit prompts to use opposites. The aim of the study was to add new evidence to recent literature on the effects of stimulating people to think about opposites when engaged in cognitive tasks and when re-structuring self-imposed representational constraints in insight problem solving – an issue which is still in its initial phase (Augustinova, 2008; Branchini et al., 2015, 2016; Gale & Ball, 2012). In particular, we were looking for new evidence regarding whether facilitation effects work better as overt and focussed strategies or rather as primers (i.e., on a non-analytical, covert level). What follows is a summary and discussion of the main results of the study.

First, in terms of the number of problems which the participants successfully solved, those in the Training condition solved more problems correctly than those in the Hint condition, independently of whether, in the initial phase, they had Overtly listed those opposite properties which might represent critical dimensions to work on in the search phase. A similar success rate was only reached in the Hint condition in the sub-condition where the participants were asked to first list all the opposite properties they could recognise before engaging in the search phase (i.e., OLH condition). The Overt Listing condition was designed to introduce a certain amount of systematicity to the participants' approach to the search phase. Therefore, overall, the results which emerged from an analysis of success rates indicate that the strategy of using opposites worked better when in association with a focussed approach due to either training or being requested to make an overt list of the opposites to be used in the subsequent search phase.

Second, neither the Hint nor Training conditions had an influence *per se* on the number of attempts. This is in line with the results of other studies regarding the effects on creative problem solving of training which aims to stimulate oppositional reasoning (Dumas, Schmidt, & Alexander, 2016). In these studies, the impact of training affected originality (as indicated by a relative measure of the number of people who have generated an idea – i.e., if fewer people come up with an idea then it is more original) rather than the number of attempts (i.e., fluency). In our study, an effect on the number of attempts was found only for those groups who found the correct solution: after Training, they did so with fewer attempts and in a shorter time as compared to the Hint condition.

Third, Overtly Listing opposites in the preliminary phase did not lead to an increase in the number of drawings done, but neither did it lead to fewer drawings due to the fact that the participants had used up part of the seven and a half minutes at their disposal making this list. However, since in effect part of the time that the participants had at their disposal was occupied with finding the opposites and writing them down, one might wonder whether the lack of difference in the overall number of drawings done should not be considered as evidence that, in the remaining time, they had in effect more ideas as compared to the participants who spent the whole seven and a half minutes doing drawings (and in spite of this, made a similar number of attempts). The impact of the listing on the number of drawings is an aspect that might be worth further investigating with an *ad hoc* experimental design. The correct solutions found in the Overt Listing conditions were, on the contrary, generally associated with longer response times as compared to the Covert Listing conditions.

Lastly, differences also emerged between the two Facilitation conditions with regard to the quality of the list of opposites produced and the use that was made of this list later on during the search phase. The lists produced in the Overt Listing Hint condition (OLH) were in a sense more "accurate" since they less frequently comprised "wrong" pairs (i.e., cases included in the nO category). They were also longer as compared to the lists produced in the Training condition. However, the pairs listed were in most cases the same as those which had been suggested by the experimenters when explaining the task. Conversely, participants in the Overt Listing Training condition (OLT) made shorter and more inaccurate lists (i.e., instances of the nO category were more frequent), but they did not limit themselves to the opposites suggested by the experimenters and tried to find new pairs. We might interpret this as an indication that they really tried to apply the *strategy* suggested and not simply re-use the opposite pairs which had already been suggested. Moreover, during the search phase, the participants also more frequently verbally reminded themselves to apply the strategy they had been instructed to use and often re-analysed the examples provided in the instructions to remember how the strategy worked. All these indications seem to us to be evidence that the participants were using the prompt as a focussed testing strategy in an incremental analytical process.

These results are interesting in terms of the debate on the facilitation factors which support insight problem solving involving non-routine problems (Gilhooly et al., 2015), that is, problems that require representational changes (insight problems) or require solutions that are "new" to the solver (creative problems). Our findings seem to indicate the effectiveness of using opposites within consciously used strategies (i.e., opposites can be effective facilitators of representational change in focussed analytical thought processes). They seem to act as a point of leverage which makes restructuring the initial mental representation of the problem more productive. The results of the present study are in line with a number of studies that have shown that the restructuring process is facilitated by instructional interventions which focus on revealing perceived barriers and tacit assumptions (e.g., Ahmed & Patrick, 2006; Ansburg & Dominowski, 2000; Chrysikou, 2006; Cunningham & MacGregor, 2008; Patrick & Ahmed, 2014; Patrick et al., 2015; Walinga et al., 2011). The reason why thinking in terms of opposites assists in this type of process is due to the dual nature of opposites which, by definition, consist of pairs of properties. By identifying the oppositional properties of the structure of the problem solvers are able to see any constraints and at the same time the means to modify them. One of the two poles on the dimension relating to an opposite pair points to the tacit barrier or assumption which is embedded in the initial representation, while the other pole is more directly conducive to potential alternatives (i.e. manipulations which restructure the representation). In this sense, opposites do not only *open up a new* space in which to search for a solution, but also provide a bounded solution space (rather than an unbounded one) by offering endpoints which delimit it. This combination of openness and boundaries fits in with the requisites of an *effective* cognitive heuristic according, for instance, to Öllinger, Jones, Danek, and Knoblich (2013).

We might also speculate that our findings can be discussed in terms of the debate on the role of Type 1 and Type 2 reasoning processes that come into play at different points in non-routine problem solving (Fleck & Weisberg, 2013; Sowden, Pringle, & Gabora, 2015; Weisberg, 2015, 2018). In particular, a strategy based on opposites seems to imply both processes (in line with what was suggested by Weisberg (2015, 2018), although this was not specifically related to opposites). The prompt given to the participants in our tasks probably operates at two stages, that is, during the generation of hypotheses and also the evaluative process. This is similar to what occurs in creative problem solving (Howard-Jones & Murray, 2003; Sowden et al., 2015). The stage involving the generation of hypotheses was activated when the participants identified all of the spatial properties implied by the verbal and graphic formulation of the problem and various possible scenarios were generated based on the opposites elicited. This phase appears to be more related to the creation of automatic associations (since opposites are primal, perceptually grounded cognitive structures). These associations are at the same time remote as they are not obvious or conservative and since opposites are *maximally* distant within a pole, they thus imply a drastic change (on this subject, see the discussion relating to continuity vs. discontinuity implied in the use of opposites in Branchini et al., 2016). All of this relates more to Type 1 processing. Conversely, the evaluative process, involves testing whether the alternatives elicited in the first (automatically driven) stage are suitable and subsequently homing in on a single solution. This corresponds to a more selective, analytical process which requires more effort (i.e., a Type 2 process) since it implies a systematic revision of all of the potential solutions which have been generated. Various studies on insight problem solving (Ball & Stevens, 2009) and creative problem solving (Barr, Pennycook, Stolz, & Fugelsang, 2015) indicate that Type 2 processes act as a catalytic in the evaluation phase rather than in the generative phase. The results of this study seem to suggest that opposites work *more effectively* when an aware, analytical process (which comes into play in the second stage) is facilitated.

The considerations which have been discussed concerning the possible role of the two types of processes are only speculative at this point in time. Further specific studies are needed to investigate the interplay between these two types of processes more deeply and to assess the facilitatory effect of thinking in terms of opposites selectively in one (or both) of these processes.

One might also ask whether the efficacy of this strategy differs depending on the degree of difficulty of the problems, for example, considering single-step problems versus multiple-step problems (Murray & Byrne, 2013). Yet another issue which is related concerns identifying the specific moment when the strategy helps the problem solver. Is it at the impasse phase (i.e., specifically after the problem solver has got stuck) or directly from the initial phase? Finally, an interesting question which has yet to be researched concerns whether this strategy is purely visuo-spatial or if there is any scope for the use of opposites in verbal insight problem solving. All food for thought in the future.

## Acknowledgements

We wish to thank the two anonymous reviewers for their careful comments which helped us to improve the quality of this version of the paper.

## Disclosure statement

No potential conflict of interest was reported by the authors.

## References

Ahmed, A., & Patrick, J. (2006). Making implicit assumptions explicit in verbal insight problem solving. In Cognitive Science Society (Ed.), *Proceeding of 28th Annual Conference of the Cognitive Science Society in Cooperation with 5th International Conference of Cognitive Science Society* (pp. 986–991). Red Hook, NY: Curran Associates, Inc. Retrieved from http://csjarchive.cogsci.rpi.edu/proceedings/2006/docs/p955.pdf.

Alexander, P. A. (2012). *The test of relational reasoning*. College Park, MD: Disciplined Reading and Learning Research Laboratory.

Alexander, P. A. (2016). Relational thinking and relational reasoning: harnessing the power of patterning. *NPJ Science of Learning*, *1*, 16004. doi:10.1038/npjscilearn.2016.4.

Alexander, P. A., Dumas, D., Grossnickle, E. M., List, A., & Firetto, C. M. (2016). Measuring relational reasoning. *The Journal of Experimental Education*, *84* (1), 119–151. doi:10.1080/00220973.2014.963216.

Ansburg, P. A. M. E. L. A. I., & Dominowski, R. O. G. E. R. I. (2000). Promoting insightful problem solving. *The Journal of Creative Behavior*, *34*(1), 30–60. doi:10.1002/j.2162-6057.2000.tb01201.x.

Antonietti, A., Colombo, B., & Pizzingrilli, P. (2011). The WCR model of creativity. From concept to application. *The Open Education Journal*, *4*(1), 80–89. doi:10.2174/1874920801104010080.

Augustinova, M. (2008). Falsification cueing in collective reasoning: Example of Wason selection task. *European Journal of Social Psychology*, *38*(5), 770–785. doi:10.1002/ejsp.532.

Ball, L., Marsh, J. E., Litchfield, D., Cook, R. L., & Booth, N. (2015). When distraction helps: Evidence that concurrent articulation and irrelevant speech can facilitate insight problem solving. *Thinking & Reasoning*, *21*(1), 76–96. doi:10.1080/13546783.2014.934399.

Ball, L. J., & Stevens, A. (2009). Evidence for a verbally-based analytic component to insight problem solving. In N. Taatgen & H. van Rijn (Eds.), *Proceedings of the 31st Annual Conference of the Cognitive Science Society*. Austin, TX: Cognitive Science Society.

Barr, D. J. (2013). Random effects structure for testing interactions in linear mixed-effects models. *Frontiers in Psychology*, *4*, 328. doi:10.3389/fpsyg.2013.00328.

Barr, D. J., Levy, R., Scheepers, C., & Tily, H. J. (2013). Random effects structure for confirmatory hypothesis testing: Keep it maximal. *Journal of Memory and Language*, *68*(3), 255–278. doi:10.1016/j.jml.2012.11.001.

Barr, N., Pennycook, G., Stolz, J. A., & Fugelsang, J. A. (2015). Reasoned connections: A dual process perspective. *Thinking and Reasoning*, *21*(1), 61–75. doi:10.1080/13546783.2014.895915.

Bates, D., Kliegl, R., Vasishth, S., & Baayen, H. (2015). Parsimonious mixed models. *Journal of Memory and Language*, arXiv:150604967. Retrived from http://arxiv.org/abs/1506.04967

Bates, D., Machler, M., Bolker, B. M., & Walker, S. C. (2015). Fitting linear mixed-effects models using lme4. *Journal of Statistic Software*, *67*, 22699. doi:10.18637/jss.v067.i01.

Bianchi, I., Burro, R., Torquati, S., & Savardi, U. (2013). The middle of the road: Perceiving intermediates. *Acta Psychologica*, *144* (1), 121–135. doi:10.1016/j.actpsy.2013.05.005.

Bianchi, I., Paradis, C., Burro, R., van de Weijer, J., Nyström, M., & Savardi, U. (2017). Identification of opposites and intermediates by eye and by hand. *Acta Psychologica*, *180*, 175–189. doi:10.1016/j.actpsy.2017.08.011.

Bianchi, I., & Savardi, U. (2018). Spatial contraries and mirrors. In Hubbard, T. (Ed.), *Spatial biases in perception and cognition*. Cambridge: Cambridge University Press.

Bianchi, I., Savardi, U., & Burro, R. (2011). Perceptual ratings of opposite spatial properties: Do they lie on the same dimension? *Acta Psychologica*, *138*(3), 405–418. doi:10.1016/j.actpsy.2011.08.003.

Bianchi, I., Savardi, U., & Kubovy, M. (2011). Dimensions and their poles: A metric and topological theory of opposites. *Language and Cognitive Processes*, *26*(8), 1232–1265. doi:10.1080/01690965.2010.520943.

Borenstein, M., Hedges, L. V., Higgins, J. P. T., & Rothstein, H. R. (2009). *Introduction to meta-analysis*. London: Wiley.

Bowden, E. M., Jung-Beeman, M., Fleck, J., & Kounios, J. (2005). New approaches to demystifying insight. *Trends in Cognitive Sciences*, *9*(7), 322–328. doi:10.1016/j.tics.2005.05.012

Bozzi, P. (2019). L'interosservazione come metodo per la fenomenologia sperimentale [Interobservation as a method for Experimental Phenomenology]. *Giornale Italiano di Psicologia, 5*, 229-239. In I. Bianchi & R. Davies (Eds.), *Paolo Bozzi's experimental phenomenology* (pp. 198–210). New York, NY: Routledge. (Original work published 1978)

Branchini, E., Bianchi, I., Burro, R., Capitani, E., & Savardi, U. (2016). Can contraries prompt intuition in insight problem solving? *Frontiers in Psychology*, *7*, 1962. doi:10.3389/fpsyg.2016.01962.

Branchini, E., Burro, R., Bianchi, I., & Savardi, U. (2015). Contraries as an effective strategy in geometrical problem solving. *Thinking & Reasoning*, *21*, 397–430. doi:10.1080/13546783.2014.

Branchini, E., Burro, R., & Savardi, U. (2009). Contraries in productive thinking. In U. Savardi (Ed.), *The perception and cognition of contraries* (pp. 203–224). Milan: McGraw-Hill.

Bröderbauer, S., Huemer, M., & Riffert, F. (2013). On the effectiveness of incidental hints in problem solving revisiting Norman Maier and Karl Duncker. *Gestalt Theory*, *35*, 349–364.

Brophy, D. R. (2006). A comparison of individual and group efforts to creatively solve contrasting types of problems. *Creativity Research Journal*, *18*(3), 293–315. doi: 10.1207/s15326934crj1803_6.

Casasola, M. (2008). The development of infants' spatial categories. *Current Directions in Psychological Science*, *17*(1), 21–25. doi:10.1111/j.1467-8721.2008.00541.x.

Casasola, M., Cohen, L. B., & Chiarello, E. (2003). Six-month-old infants' categorization of containment spatial relations. *Child Development*, *74*(3), 679–693. doi:10.1111/1467-8624.00562.

Chronicle, E. P., MacGregor, J. N., & Ormerod, T. C. (2004). What makes an insight problem? The role of heuristics, goal conception and solution recoding in knowledge-lean problems. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *30*, 14–27. doi:10.1037/0278-7393.30.1.14.

Chronicle, E. P., Ormerod, T. C., & MacGregor, J. N. (2001). When insight just won't come: The failure of visual cues in the nine-dot problem. *Quarterly Journal of Experimental Psychology, 54A*, *54*(3), 903–919. doi:10.1080/713755996.

Chrysikou, E. G. (2006). When shoes become hammers: Goal-derived categorization training enhances problem solving performance. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *33*, 935–942. doi:10.1037/0278-7393.32.4.935.

Collett, D. (2003). *Modelling survival data in medical research*. London: Chapman Hall.

Cunningham, J. B., & MacGregor, J. N. (2008). Training insightful problem solving: Effects of realistic and puzzle-like contexts. *Creativity Research Journal*, *20*, 291–296. doi:10.1080/10400410802278735.

Danek, A. H., & Wiley, J. (2017). What about false insights? Deconstructing the Aha! experience along its multiple dimensions for correct and incorrect solutions separately. *Frontiers in Psychology*, *7*, 2077. doi:10.3389/fpsyg.2016.02077.

De Bono, E. (1969). *The mechanism of mind*. New York, NY: Penguin.

Dominowski, R. L., & Dallob, P. (1995). Insight and problem solving. In R. J. Sternberg & J. E. Davidson (Eds.), *The nature of insight* (pp. 33–62). Cambridge, MA: MIT Press.

Dow, G. T., & Mayer, R. E. (2004). Teaching students to solve insight problems: Evidence for domain specificity in creativity training. *Creativity Research Journal*, *16*(4), 389–402. doi:10.1080/10400410409534550.

Dumas, D., & Alexander, P. A. (2016). Calibration of the test of relational reasoning. *Psychological Assessment*, *28*(10), 1303–1318. doi:10.1037/pas0000267.

Dumas, D., Schmidt, L., & Alexander, P. (2016). Predicting creative problem solving in engineering design. *Thinking Skills and Creativity*, *21*, 50–66. doi:10.1016/j.tsc.2016.05.002.

Duncker, K. (1945). On problem-solving. *Psychological Monographs*, *58* (5), i–113. doi: 10.1037/h0093599.

Evans, J. S. B. T., & Stanovich, K. E. (2013a). Dual-process theories of higher cognition: Advancing the debate. *Perspectives on Psychological Science*, *8*(3), 223–241. doi: 10.1177/1745691612460685.

Evans, J. S. B. T., & Stanovich, K. E. (2013b). Theory and metatheory in the study of dual processing: Reply to comments. *Perspectives on Psychological Science*, *8*(3), 263–271. doi:10.1177/1745691613483774.

Fedor, A., Szathmáry, E., & Öllinger, M. (2015). Problem solving stages in the five square problem. *Frontiers in Psychology*, *6*, 1050. doi:10.3389/fpsyg.2015.01050.

Fleck, J. I., & Weisberg, R. W. (2004). The use of verbal protocols as data: An analysis of insight in the candle problem. *Memory & Cognition*, *32*, 990–1006. doi:10.3758/BF03196876.

Fleck, J. S., & Weisberg, R. W. (2013). Insight versus analysis: Evidence for diverse methods in problem solving. *Journal of Cognitive Psychology*, *25*(4), 436–463. doi:10.1080/20445911.2013.779248.

Fox, J. (2003). Effect displays in R for generalised linear models. Retrieved from http://psfaculty.ucdavis.edu/bsjjones/effectdisplays.pdf

Fox, J., & Weisberg, S. (2011). *An R companion to applied regression* (2nd ed.). Thousand Oaks, CA; Sage.

Gale, M., & Ball, L. J. (2009). Exploring the determinants of dual goal facilitation in a rule discovery task. *Thinking & Reasoning*, *15*, 294–315. doi:10.1080/13546780903040666.

Gale, M., & Ball, L. J. (2012). Contrast class cues and performance facilitation in a hypothesis testing task: Evidence for an iterative counterfactual model. *Memory & Cognition*, *40*, 408–419. doi:10.3758/s13421-011-0159-z.

Gärdenfors, P. (2000). *Conceptual spaces: On the geometry of thought*. Cambridge, MA: MIT Press.

Gibson, J. M., Dhuse, S., Hrachovec, L., & Grimm, L. R. (2011). Priming insight in groups: Facilitating and inhibiting solving an ambiguously worded insight problem. *Memory & Cognition*, *39*, 128–146. doi:10.3758/s13421-010-0014-7.

Gick, M. L., & Holyoak, K. J. (1980). Analogical problem solving. *Cognitive Psychology*, *12*(3), 306–355. doi:10.1016/0010-0285(80)90013-4.

Gilhooly, K., Ball, L., & Macchi, L. (2015). Insight and creative thinking processes: Routine and special. *Thinking & Reasoning*, *21*(1), 1–4. doi:10.1080/13546783.2014.966758.

Gilhooly, K. J., & Fioratou, E. (2009). Executive functions in insight versus non-insight problem solving: An individual differences approach. *Thinking & Reasoning*, *15*(4), 355–376. doi:10.1080/13546780903178615.

Grant, E. R., & Spivey, M. J. (2003). Eye movements and problem solving: guiding attention guides thought. *Psychological Science*, *14*(5), 462–466. doi:10.1111/1467-9280.02454.

Harrington, D. P., & Fleming, T. R. (1982). A class of rank test procedures for censored survival data. *Biometrika*, *69*(3), 553–566. doi:10.2307/2335991.

Hattori, M., Sloman, S. A., & Orita, R. (2013). Effects of subliminal hints on insight problem solving. *Psychonomic Bulletin & Review*, *20*, 790–797. doi:10.3758/s13423-013-0389-0.

Howard-Jones, P. A., & Murray, S. (2003). Ideational productivity, focus of attention and context. *Creativity Research Journal*, *15*(2), 153–166. doi:10.1207/S15326934CRJ152&3_07.

Jablansky, S., Alexander, P. A., Dumas, D., & Compton, V. (2016). Developmental differences in relational reasoning among primary and secondary school students. *Journal of Educational Psychology*, *108*(4), 592–608. doi:10.1037/edu0000070.

Jablansky, S., Alexander, P. A., & Singer, L. (2016). Examining the relational reasoning capabilities of elementary and middle-school students with learning needs. Paper presented at the annual meeting of the American Educational Research Association. Washington, DC.

Kaplan, E. L., & Meier, P. (1958). Nonparametric estimation from incomplete observations. *Journal of the American Statistical Association*, *53*(282), 457–485. doi:10.1080/01621459.1958.10501452.

Kaplan, C. A., & Simon, H. A. (1990). In search of insight. *Cognitive Psychology*, *22*, 373–419. doi:10.1016/0010-0285(90)90008-R.

Katona, G. (1940). *Organizing and memorizing: Studies in the psychology of learning and teaching*. New York, NY: Columbia University.

Köhler, W. (1969). *The task of Gestalt psychology*. Princeton, NJ: Princeton University Press.

Kershaw, T. C., Flynn, C. K., & Gordon, L. T. (2013). Multiple paths to transfer and constraint relaxation in insight problem solving. *Thinking & Reasoning*, *19*(1), 96–136. doi:10.1080/13546783.2012.742852.

Kershaw, T. C., & Ohlsson, S. (2004). Multiple causes of difficulty in insight: The case of the nine-dot problem. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *30*, 3–13. doi:10.1037/0278-7393.30.1.3.

Knoblich, G., Ohlsson, S., Haider, H., & Rhenius, D. (1999). Constraint relaxation and chunk decomposition in insight problem solving. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *25*, 1534–1555. doi:10.1037/0278-7393.25.6.1534.

Laughlin, P. R., Bonner, B. L., & Miner, A. G. (2002). Groups perform better than best individuals on letters-to-numbers problems. *Organizational Behavior and Human Decision Processes*, *88*(2), 605–620. doi:10.1016/S0749-5978(02)00003-1.

Laughlin, P. R., & Futoran, G. C. (1985). Collective induction: Social combination and sequential transition. *Journal of Personality and Social Psychology*, *48*(3), 608–613. doi:10.1037/0022-3514.48.3.608.

Lenth, R. V. (2016). Least-squares means: The R package lsmeans. *Journal of Statistical Software*, *69*, 3305.

Litchfield, D., & Ball, J. L. (2011). Using another's gaze as an explicit aid to insight problem solving. *The Quarterly Journal of Experimental Psychology*, *64*(4), 649–656. doi:10.1080/17470218.2011.558628.

Maier, N. R. F. (1930). Reasoning in humans. On direction. *Journal of Comparative Psychology*, *10*(2), 115–143. doi:10.1037/h0073232.

Maier, N. R. F. (1931). Reasoning in humans: II. The solution of a problem and its appearance in consciousness. *Journal of Comparative Psychology*, *12*(2), 181–194. doi:10.1037/h0071361.

Moshman, D., & Geil, M. (1998). Collaborative reasoning: Evidence for collective rationality. *Thinking and Reasoning*, *4*, 231–248. doi:10.1080/135467898394148.

Murray, M. A., & Byrne, R. M. J. (2013). Cognitive change in insight problem solving: Initial model errors and counterexamples. *Journal of Cognitive Psychology*, *25*(2), 210–219. doi:10.1080/20445911.2012.743986.

Newell, A., & Simon, H. (1972). *Human problem solving*. Englewood Cliffs, NJ: Prentice Hall.

Ohlsson, S. (1984a). Restructuring revisited: I. Summary and critique of the Gestalt theory of problem solving. *Scandinavian Journal of Psychology*, *25*(1), 65–78. doi: 10.1111/j.1467-9450.1984.tb01001.x.

Ohlsson, S. (1984b). Restructuring revisited: II. An information processing theory of restructuring and insight. *Scandinavian Journal of Psychology*, *25*(2), 117–129. doi: 10.1111/j.1467-9450.1984.tb01005.x.

Ohlsson, S. (1992). Information-processing explanations of insight and related phenomena. In M. T. Keane, & K. J. Gilhooly (Eds.), *Advances in the psychology of thinking* (Vol.1, pp. 1–44). New York, NY: Harvester Wheatsheaf.

Ohlsson, S. (2011). *Deep learning: How the mind overrides experience*. Cambridge: Cambridge University Press.

Öllinger, M., Jones, G., Danek, A. H., & Knoblich, G. (2013). Cognitive mechanisms of insight: The role of heuristics and representational change in solving the Eight-Coin problem. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *39*, 931–939. doi:10.1037/a0029194.

Öllinger, M., & Knoblich, G. (2009). Psychological research on insight problem solving. In H. Atmanspacher & H. Primas (Eds.), *Recasting reality* (pp. 275–300). Berlin, Germany: Springer. doi:10.1007/978-3-540-85198-1_14.

Ormerod, T. C., MacGregor, J. N., & Chronicle, E. P. (2002). Dynamics and constraints in insight problem solving. *Journal of Experimental Psychology: Learning, Memory & Cognition*, *28*, 791–799. doi:10.1037//0278-7393.28.4.791.

Patrick, J., & Ahmed, A. (2014). Facilitating representation change in insight problems through training. *Journal of Experimental Psychology: Learning, Memory & Cognition*, *40*, 532–543. doi:10.1037/a0034304.

Patrick, J., Ahmed, A., Smy, V., Seeby, H., & Sambrooks, K. (2015). A cognitive procedure for representation change in verbal insight problems. *Journal of Experimental Psychology: Learning, Memory & Cognition*, *41*, 746–759. doi:10.1037/xlm0000045.

Perkins, D. N. (2000). *The Eureka effect. The art and logic of breakthrough thinking*. New York, NY: Norton.

Quinn, P. C., Cummins, M., Kase, J., Martin, E., & Weissman, S. (1996). Development of categorical representations for above and below spatial relations in 3- to 7-month-old infants. *Developmental Psychology*, *32*(5), 942–950. doi:10.1037/0012-1649.32.5.942.

Rothenberg, A. (1973a). Word association and creativity. *Psychological Reports*, *33*(1), 3–12. doi:10.2466/pr0.1973.33.1.3.

Rothenberg, A. (1973b). Opposite responding as a measure of creativity. *Psychological Reports*, *33*(1), 15–18. doi:10.2466/pr0.1973.33.1.15.

Rothenberg, A. (1996). The Janusian process in scientific creativity. *Creativity Research Journal*, *9*(2-–3), 207–232. doi:10.1080/10400419.1996.9651173.

Rothenberg, A. (2001). Bipolar illness, creativity, and treatment. *Psychiatric Quarterly*, *72*(2), 131–147. doi:10.1023/A:1010367525951.

Rothenberg, A., & Sobel, R. S. (1981). Effects of shortened exposure time on the creation of literary metaphors as stimulated by superimposed versus separated visual images. *Perceptual and Motor Skills*, *53* (3), 1007–1009. doi:10.2466/pms.1981.53.3.1007.

Sak, U. (2009). Creative Reversal Act: Teaching the ways creators think. *Gifted Education International*, *1*, 5–13. doi:10.1177/026142940902500103.

Sak, U., & Oz, O. (2010). The effectiveness of the Creative Reversal Act (CREACT) on students' creative thinking. *Thinking Skills and Creativity*, *5*(1), 33–39. doi:10.1016/j.tsc.2009.09.004.

Savardi, U., & Bianchi, I. (2009). The spatial path to contrariety. In U. Savardi (Ed, *The perception and cognition of contraries* (pp. 63–92). Milan, IT: Mc-Graw Hill.

Schooler, J. W., Ohlsson, S., & Brooks, K. (1993). Thoughts beyond words: When language overshadows insight. *Journal of Experimental Psychology: General*, *122*(2), 166–183. doi:10.1037/0096-3445.122.2.166.

Seifert, C. M., Meyer, D. E., Davidson, N., Patalano, A. L., & Yaniv, I. (1995). Demystification of cognitive insight: Opportunistic assimilation and the prepared-mind perspective. In R. J. Sternberg & J. E. Davidson (Eds.), *The nature of insight* (pp. 65–124). Cambridge, MA: MIT Press.

Siegler, R. S. (2000). Unconscious insights. *Current Directions in Psychological Science*, *9*(3), 79–83. doi:10.1111/1467-8721.00065.

Singmann, H., Bolker, B. M., & Westfall, J. (2017). AFEX: Analysis of factorial experiments. R Package Version 0.16-1.R Foundation for Statistical Computing, Ed.; Vienna. Available online: http://CRAN.R-project.org/package=afex (accessed on 26 January 2017).

Sobel, R. S., & Rothenberg, A. (1980). Artistic creation as stimulated by superimposed versus separated visual images. *Journal of Personality and Social Psychology*, *39*(5), 953–961. doi:10.1037/0022-3514.39.5.953.

Sowden, P. T., Pringle, A., & Gabora, L. (2015). The shifting sands of creative thinking: Connections to dual-process theory. *Thinking & Reasoning*, *21*, 40–60. doi:10.1080/13546783.2014.885464.

Therneau, T. M., & Grambsch, P. M. (2000). *Modeling survival data: Extending the Cox model*. New York: Springer.

Walinga, J., Cunningham, J. B., & MacGregor, J. N. (2011). Training insight problem solving through focus on barriers and assumptions. *The Journal of Creative Behavior*, *45*(1), 47–58. doi:10.1002/j.2162-6057.2011.tb01084.x.

Webb, M. E., Little, D. R., & Cropper, S. J. (2016). Insight is not in the problem: Investigating insight in problem solving across task types. *Frontiers in Psychology*, *7*, 1424. doi:10.3389/fpsyg.2016.01424.

Weisberg, R. W. (2015). Toward an integrated theory of insight in problem solving. *Thinking & Reasoning*, *21*, 5–39. doi:10.1080/13546783.2014.886625.

Weisberg, R. W. (2018). Problem solving. In L. J. Ball & V. A. Thompson (Eds.), *The Routledge International Handbook of Thinking and Reasoning* (pp. 1239–1274). New York, NY: Routledge.

Wertheimer, M. (1945). *Productive thinking*. Harper, New York. First appearance Über Schluzbprozesse in productiven Denken. (Original work published 1919)

Wiley, J., & Jarosz, A. (2012). Working memory capacity, attentional focus, and problem solving. *Current Directions in Psychological Science*, *21*(4), 258–262. doi:10.1177/0963721412447622.

World Medical Association. (2013). World Medical Association Declaration of Helsinki: Ethical principles for medical research involving human subjects. *JAMA*, *310*, 2191–2194. doi:10.1001/jama.2013. 281053.

## Appendix 1

### *Summary of the instructions given to participants in the training and hint conditions*

In the TRAINING condition, participants were told that "Often the solution to visuospatial problems can be found by focussing on the perceptual properties of the problem and transforming some of these properties into their opposites. We will show you some examples to illustrate what we mean. (…) Note that the FIRST transformation that comes to mind may not necessarily be the one that leads to success. But by applying this strategy, it helps us to overcome a block in our reasoning process and to discover the critical transformation which will lead to the solution."

Three examples were provided (Table 3), the Parallelogram problem, the Nine dots problem and the Altar window problem.

The spatial features of the problem to be solved were described to the participants in detail. What follows are portions of the instructions which were then read out.

For the Parallelogram problem: "Operationally speaking, a strategy that we can apply is to start with the evident properties of the figure and then reason by seeing if it is possible to transform these into their opposites.

- - The figure has long horizontal sides. Can I find the solution by making them vertical? (In this case no, I can't.)
- - The figure has parallel sides. Can I find the solution by making them diverge or converge? (In this case, no I can't.)
- - The figure leans to one side. Can I find the solution by straightening the figure? (In this case, yes I can)
- - The long sides are misaligned. Can I find the solution by aligning them? (In this case, yes I can)".

For the Nine dot problem: "Again, operationally speaking, a strategy that we can apply is to start from the evident properties of the figure and then reason in terms of opposites.

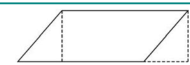
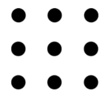
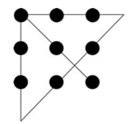
- - I see horizontal and vertical rows of dots. Can I find the solution by drawing oblique lines as well as horizontal and vertical lines? (In this case yes, I can)
- - The dots are inside the square. Can I find the solution by drawing lines which go outside the square? (In this case, yes I can)".

For the Altar window problem: "Let's try to apply the strategy that we have learnt.

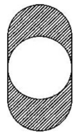
- - The figure is curvilinear. What if the solution requires us to draw straight lines?
- - The circle in the middle is empty. What if the solution involves filling that empty space?
- - The figure develops vertically above and below the circle. What if the solution requires us to draw horizontal lines?
- - The area to be covered is united. What if the solution requires us to divide it into parts?"

In the HINT condition, the instructions required the participants to first identify all of the spatial properties mentioned in the text of the problem or shown in the figure and to transform each of them into their corresponding opposite property. The same three example problems were provided to exemplify the hint. The spatial properties characterising each example problem were described by the experimenter and the pairs of opposite properties that are summarised in the last column of Table 3 are mentioned.

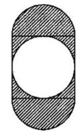
**Table 3.** (in Appendix 1) The three example problems used to explain the strategy in the hint and training conditions, with the solution and the opposites for each problem.

| Problem | Initial representation | Solution | Opposites mentioned in the explanation of the strategy |
|---|---|---|---|
| Parallelogram |  The task is to discover how to calculate the area of the figure and explain why this is a correct method |  This can be done by drawing two perpendicular, vertical lines down from the top two vertices of the parallelogram. If we move the triangle that now stands out on the left so that it covers the triangle that seems to be missing on the right, we obtain a rectangle. It is then relatively easy to calculate the area of the figure. | Long—Short Horizontal—Vertical Parallel—Divergent/Convergent Aligned—Misaligned Leaning—Upright Left—Right |
| Nine dots |  The task is to connect all the dots using four straight lines, without taking the pencil from the paper and without passing twice over the same dot. |  We tend to think we must join the dots either horizontally or vertically while remaining inside the square. Instead the solution requires us to draw lines that are oblique as well as horizontal or vertical and use lines which also go outside the square. | Inside—Outside Horizontal—Vertical Oblique—Straight |

| Altar window | | | Curved—Rectilinear Vertical—Horizontal Empty—Full United—Separated |
|---|---|---|---|



Workers are painting and decorating the inner walls of a church. There is a circular window a little above the altar. As a decoration, the painters have been asked to draw two vertical lines tangentially to and of the same height as the circular window; they are then to add half circles above and below, closing in the figure. This area between the lines and the window is to be covered with gold. The problem requires to find out how to calculate the area to be covered with gold (the shaded area).



The area to be covered with gold coincides with the area of the central circle plus the four pieces outside the circle at the corners of the square. Therefore the curvilinear area to be covered with gold coincides exactly with the area of the square.